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Designing Canopy Walkways: Engineering Calculations for Building Canopy Access Systems with Cable-Supported Bridges

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Abstract. Canopy access systems that include cable-supported walkway bridges are being built around the world to study forest canopy ecosystems. The ''canopy walkways'' considered here require less physical effort on the part of researchers than do rope climbing techniques. Such systems also facilitate collaboration between several researchers moving laterally through the canopy. The physics of building hanging structures, such as canopy bridges, needs to be understood and utilized to design a safe, long-lasting structure. An interactive computer program was developed that employs catenary curve equations and nested root extraction algorithms to calculate construction parameters. The use of this program has accelerated the design of several structures built in the canopy of both temperate and tropical forests.

Key words: Canopy, walkway construction, temperate forest, tropical forest

Introduction

Research in the canopies of forests (Lowman & Wittman 1996) has been limited by the logistical constraints of safe and easy access (reviewed in Mitchell 1982, Moffet & Lowman 1995). Of the many methods used to gain access to the heights, only a few allow several people to be in close proximity or in the canopy for long time periods. Observation platforms and canopy bridges are used to overcome these limitations, but the design and costs of many such structures often exceed research budgets. Compared to suspension bridges, the canopy bridges discussed here are lighter, easier to construct and install, and can be placed higher in the canopy. Shortening the design phase while maintaining safety standards may reduce the cost.

Walkway Systems

A canopy walkway system typically incorporates some combination of platform(s), bridge(s), and a means of access (Lowman & Bouricius 1995). A canopy bridge as discussed here consists of an overhead safety cable, two handrail cables, and a treadway (Figure 1). The treadway consists of two strong and flexible supporting cables with slender traversing treads, separated by spacers, distributed along their entire lengths. The ends of each cable are attached to large trees that are stabilized with guy cables. The treadway hangs freely and assumes a special shape called a catenary. In 1691, Johann Bernouilli used principles of physics to describe the exact shape formed by a chain suspended between two points (Dunham 1990). Any such freely hanging entity, if it is perfectly flexible and has uniform weight per unit length, will assume a catenary shape. The treadways described here weigh very nearly a constant amount per unit length, the treads are numerous, and the cables are quite flexible. Deviations from a catenary shape, therefore, are quite small. Consequently, mathematical modeling is both feasible and trustworthy (USSC 1959).

Design Considerations

To design and construct a walkway, one must know the attributes of all the materials used, such as the tensile strength of the cables and the weights of all component parts. The treadway safety factor is defined as the minimum breaking

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FIGURE 1. Photograph of walkway during construction at The EcoTarium, Worcester, MA.

strength of the supporting cables divided by the cable tension when the load equals its maximum allowed value. For our treadways, we prescribe safety factors of 5 or higher (WRTB 1993). To achieve a prescribed safety factor, the tension on the cables must either be measured or calculated. Because measuring cable tension during construction would be costly, time-consuming, and impractical, it is preferable to calculate the tensions. To permit easy negotiation, the slope of the treadway should not be too large. To accomplish this, we prescribe an upper limit of 25° for the angle from the horizontal, at any place on our treadways. In considering a first choice for supporting cables, a designer may find calculations revealing that the required safety factor cannot be met, or that the angles at the ends of the treadway would become too large. If so, then higher tensile-strength support cables must be used. This means heavier, thicker, and more expensive cables. A reevaluation of every part of the whole walkway system must then be made to assess the effects of using stronger cables. Different choices for eyebolts, guy cables, or even trees might be necessary.

This design process takes into account ergonomic and economic factors in addition to the dimensional ones. Design specifications are considered and resolved prior to ordering the structural components. This minimizes the need for field modifications. With the final specifications determined, the necessary materials are purchased and transported to the site. Then the actual construction of the canopy access system begins.

Construction Considerations

To specify a particular treadway catenary requires two parameters, one of which is the horizontal distance between the two ends, defined as the span. The other parameter may be the length along the catenary, the angle at the ends, or the sag. Specifying any one of them determines the curvature coefficient, *A*. The others are then easily calculated. The angle at the attachment point is extremely difficult to measure accurately, as is the length along the treadway catenary. For a 50-foot span with an initial sag of 3 feet, an error of 1 inch in the measured sag is equivalent to a 0.37° error in the angle or to a 0.027-ft error in the measured catenary length (TABLE 1). The most feasible way to assure during construction that a treadway meets its design specifications is to measure the sag. A line of sight method using a portable laser level works well. Repeated measurements on a treadway having a 50-foot span vary by less than an inch. The sag is changed during construction by varying the length of the support cables. This is ac-

TABLE 1. Calculated values for a level 50-foot span treadway, weighing 8.9 pounds-per-foot and supported by two ⅜″ stainless steel cables. Each cable has a minimum-breaking-strength of 12,000 pounds.

Sag (ft) (stage 1)	Curvature A	Angle θ	Tension (lbs) per cable	Safety factor	Catenary length (ft)
1.00	312.7	4.58°	1396	8.60	50.053
1.25	250.3	5.72°	1119	10.72	50.083
1.50	208.6	6.85°	935	12.83	50.120
1.75	178.9	7.98°	803	14.93	50.163
2.00	156.7	9.11°	705	17.00	50.213
2.25	139.3	10.23°	630	19.05	50.269
2.50	125.4	11.35°	569	21.07	50.332
2.75	114.1	12.46°	520	23.07	50.401
3.00	104.7	13.56°	479	25.04	50.477
3.25	96.7	14.65°	445	26.97	50.559
3.50	89.9	15.74°	416	28.87	50.647
3.75	84.0	16.82°	390	30.73	50.742
4.00	78.8	17.88°	368	32.56	50.843

complished by adjusting the cables through the cable clamps and eye bolts at one end of the treadway. This procedure is repeated until the measured sag attains its prescribed design value.

CATENARY EQUATIONS

We first consider a mathematical catenary and then progress to a physical treadway, one that has a catenary shape but also has a known constant weight per foot. We categorize a catenary or treadway to be ''level'' when the two ends are in the same horizontal plane (FIGURES 2, 3) and ''inclined'' (FIGURE 4) when they are not. Specific values of the Span, Sag, and Incline are required to calculate the value of the ''curvature'' coefficient, A, for a catenary.

These are the hyperbolic equations of a catenary (Anton 1984):

$$y(x) = A\cdot\cosh(x/A) \quad (1)$$

$$\text{Tangent } \theta(x) = dy/dx = \sinh(x/A) \quad (2)$$

$$L(x) = S\cdot\sinh(x/A) \quad (3)$$

Equation 1 is the fundamental equation of a catenary. In equation 2, tangent $\theta(x)$ is the slope of the catenary, and $\theta(x)$ is the angle from the horizontal, at x. In equation 3, $L(x)$ is the arc length along the catenary from the $x = 0$ point to point x (see APPENDIX).

A real physical catenary, as opposed to a mathematical one, possesses weight. This weight creates a tension, $T(x)$, in the supporting cables of the treadway. By taking into consideration the static equilibrium status of such a catenary and its assumed perfect flexibility and uniformity, the following three properties can be deduced:

1. The vertical component of $T(x)$ is always equal to the summed weight of the catenary from the $x = 0$ point to point x.
2. The horizontal component of $T(x)$, however, remains constant throughout.
3. The ratio of the vertical to the horizontal component at any point is equal to the tangent at that point.

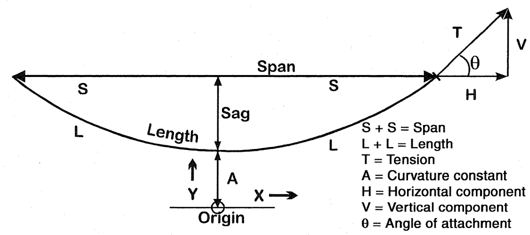

FIGURE 2. A level symmetrical treadway with its various parts labeled.

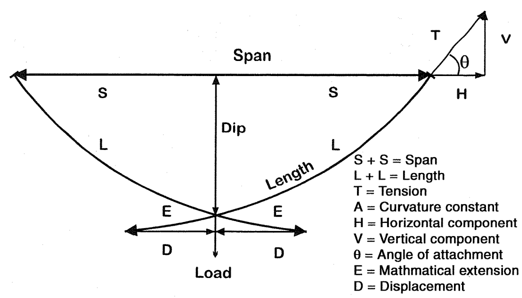

FIGURE 3. A level symmetrical treadway with a load placed at its center. The equations pertinent to this situation must include an x displacement of D as shown here.

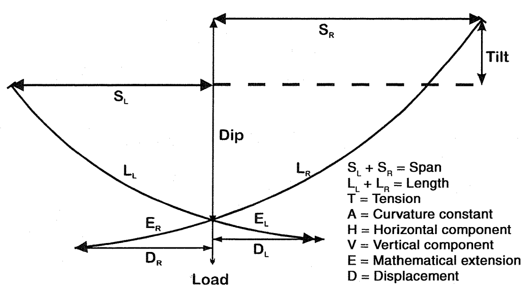

FIGURE 4. A treadway that is inclined and has a load placed on it at some arbitrary horizontal position, S_L.

In the following derivations, V denotes the vertical component and H, the horizontal component of the tension T. W denotes the weight per unit length. Making use of equations 2 and 3, and the three properties of real catenaries, we derive equations 4, 5, and 6:

TABLE 2. Calculated values for a level 50-foot span treadway, weighing 8.9 pounds-per-foot and holding a 1000-pound load. The treadway is supported by two ⅜″ stainless steel cables, each with a minimum-breaking-strength of 12,000 pounds.

Sag (ft) (stage 1)	Dip (ft) (stage 2)	Curvature A	Angle θ	Tension (lbs) per cable	Safety factor	Catenary length (ft)
1.00	1.15	1495.6	3.11°	6665	1.80	50.053
1.25	1.44	1196.6	3.88°	5337	2.25	50.083
1.50	1.72	997.4	4.66°	4453	2.70	50.120
1.75	2.01	855.1	5.43°	3822	3.14	50.163
2.00	2.30	748.4	6.20°	3350	3.58	50.213
2.25	2.58	665.4	6.97°	2983	4.02	50.269
2.50	2.87	599.1	7.73°	2690	4.46	50.332
2.75	3.16	544.8	8.50°	2451	4.90	50.401
3.00	3.44	499.6	9.26°	2253	5.33	50.477
3.25	3.73	461.4	10.06°	2085	5.76	50.559
3.50	4.01	428.6	10.78°	1942	6.18	50.647
3.75	4.30	400.2	11.52°	1818	6.60	50.742
4.00	4.58	375.4	12.26°	1710	7.02	50.843

$$V(x) = W \cdot L(x) = W \cdot A \cdot \sinh(x/A) \qquad (4)$$

$$H(x) = V(x)/\text{tangent } \theta(x)$$

$$= W \cdot A \cdot \sinh(x/A)/\sinh(x/A) = W \cdot A \qquad (5)$$

$$T(x)^2 = V(x)^2 + H(x)^2$$

$$= W^2 \cdot A^2 \cdot [1 + \sinh^2(x/A)]$$

$$= W^2 \cdot A^2 \cdot \cosh^2(x/A) \qquad \text{(see APPENDIX)}$$

$$T(x) = W \cdot A \cdot \cosh(x/A) \qquad (6)$$

TREADWAY CALCULATIONS

The three treadways that we next consider involve increasingly complex examples. They show how catenary equations and component properties are used to calculate treadway attributes. The ability to do these calculations rapidly facilitates the design and subsequent materials selection of our freely hanging treadways.

Symmetrical Loadless Treadway

A level symmetrical treadway with its various parts labeled is shown in FIGURE 2. As is readily seen, S is half the span, L is half the length, and the value of y at x = zero is A. Using equations 3, 4, 5, and 6, we obtain the following by substitution:

$$\text{Length} = 2 \cdot L = 2 \cdot A \cdot \sinh(S/A)$$

$$\text{Sag} = A \cdot \cosh(S/A) - A$$

$$V(S) = W \cdot A \cdot \sinh(S/A)$$

$$H = W \cdot A$$

$$T(S) = W \cdot A \cdot \cosh(S/A)$$

These equations were employed to calculate the values in TABLE 1.

Symmetrical Loaded Treadway

Now consider what happens when a load is placed on the center of a level symmetrical treadway (FIGURE 3). This position is the "worst case" position that maximizes the tension on the supporting cables. The middle of the treadway naturally sinks, but the values of S and L do not change, and the left-right symmetry still holds. The treadway shown consists of two real parts, each labeled L, and two abstract mathematical extensions, each labeled E. The length of E depends directly on the weight of the load. If, for example, the weight of the load is equal to the weight of the original treadway, then E has the same length as L. This follows from the property noted pertaining to the vertical component of the tension. The equations relevant to FIGURE 3 must include an x displacement, D, as shown here:

$$2 \cdot E = \text{load}/W$$

$$E = A \cdot \sinh(D/A)$$

$$L = A \cdot \sinh[(S + D)/A] - E$$

$$\text{Tangent}(\theta) = \sinh[(S + D)/A]$$

$$\text{Dip} = A \cdot \cosh[(S + D)/A] - A \cdot \cosh(D/A)$$

$$T(S + D) = A \cdot \cosh[(S + D)/A]$$

This tension, $T(S + D)$, at the attachment point is the one used to calculate the safety factor.

These equations were employed to calculate the values in TABLE 2.

Asymmetrical Loaded Treadway

The most complex example is a treadway that has an incline (FIGURE 4) and has a load placed on it at the horizontal position, S_L that maximizes the cable tensions. Although each side has its own parameters, the curvature coefficient, A, is the same for both sides. This is proved by equating the horizontal component of the tension on the left side, to that on the right. We know that this is a true relationship because a real catenary always adopts a static equilibrium shape. From equation 5, we have $W \cdot A_L = W \cdot A_R$. Since the W on the left side is the same W as that on the right, a cancellation proves the two A's equal. The equations pertinent to FIGURE 4 are the following:

$$E_L + E_R = \text{load}/W$$

$$E_L = A \cdot \sinh(D_L/A)$$

$$E_R = A \cdot \sinh(D_R/A)$$

$$L_L = A \cdot \sinh[(S_L + D_L)/A] - E_L$$

$$L_R = A \cdot \sinh[(S_R + D_R)/A] - E_R$$

$$\text{Dip}_L = A \cdot \cosh[(S_L + D_L)/A] - A \cdot \cosh(D_L/A)$$

$$\text{Dip}_R = A \cdot \cosh[(S_R + D_R)/A] - A \cdot \cosh(D_R/A)$$

$$T_L(S_L + D_L) = A \cdot \cosh[(S_L + D_L)/A]$$

$$T_R(S_R + D_R) = A \cdot \cosh[(S_R + D_R)/A]$$

The two tensions differ, and the larger is the one used to calculate the safety factor. The "worst case" load position, although not obvious, is easily found.

It is apparent, in both TABLE 1 and TABLE 2, that a small decrease in the treadway length correlates with an increasingly large increase in the cable tensions as the length shrinks. If the two trees supporting the treadway sway, the span could increase. That motion could cause significant increases in the tensions. The canopy walkways discussed here constrain movements of the supporting trees. Guy cables stabilize both trees, and five cables, which connect the two trees, tend to synchronize their movements. No deleterious incidents due to tree motions have ever occurred in our canopy walkways.

MODELING SCENARIO

The mathematical modeling is performed in two stages. Calculations for a loadless treadway, like that of FIGURE 2, are performed during the first stage. For a given span and sag, the value of the resulting catenary length is determined. The length so calculated remains invariant during the second stage. The second stage calculates the effect of placing a specified load on the treadway. The load is placed at the position that maximizes the tension on the supporting cables. That tension, divided into the minimum breaking strength of the cables, yields the safety factor. These two stages are repeated with different values for the sag until the required safety factor is achieved. That particular sag value is the one that must be attained during the actual construction of the treadway.

An interactive program (Halvorson & Rygmyr 1991) was written that requires the following input parameters to be specified, as the program pauses and asks for each one by name:

- Span — Horizontal distance between attachment points.
- Sag — Vertical treadway displacement while loadless.
- Incline — Vertical distance between attachment points. Defaults to zero.
- W — Treadway weight per foot.
- MBS — Minimum breaking strength of a single support cable.
- Load — Total weight of people and equipment allowed on the bridge.
- S_R & S_L — Horizontal position of the load along the span. These two parameters are only required for asymmetric treadways.

For convenience and comparison purposes, the following two items also are requested; convenient default values are available:

File Name—Stores inputs and resulting calculated values on the hard drive.

Precision—Prescribes number of significant digits in the calculations. Defaults to five.

The program calculates values for the treadway length, the dip, the cable tension at the attachment points, and the resultant safety factor. These values are displayed on the computer screen. Additional parameters are calculated to confirm the operation of the proper root extractions and to indicate that the program is functioning correctly. Text files of all input parameters and output values are automatically created and stored for later review and printing.

A loadless level treadway (FIGURE 2) requires a first estimate for A, the curvature coefficient. The program then finds the value of A corresponding to the input value of the sag. The program contains a root extraction subroutine that employs Newton's method (Arfken 1985) to locate that root.

For a level treadway carrying a load (FIGURE 3), the program first calculates the length of the loadless catenary. Then it employs two nested root extraction subroutines, one nested within the other, to find A and D, such that the product of E and W equals half the load.

When the treadway is inclined, or when the load is not placed on the center of a level treadway, an asymmetry occurs as depicted in FIGURE 4. For such circumstances, the program employs three nested root extracting subroutines that find A, D_L, and D_R. The program then proceeds to calculate the rest of the output parameters.

All root extraction algorithms require a first estimate of their root values. That value is then stepwise modified until it meets a specified precision. Newton's root extraction method requires a value of the tangent of the root equation at every step of the iteration. Calculations employing a $\Delta Y/\Delta X$ approximation for the tangent sometimes created divide-by-zero or out-of-bound computer errors that restricted the program's functioning. Eventually a way was found to formulate all of the root equations in ways that were analytically differentiable. The first derivative of the root equations, which are the tangents, then became available. These formulations proved more robust. The program accepts a wide range of input values, limited only by the practical values of treadway weight and cable strength.

CONCLUSION

Use of catenary curve equations and nested root extraction algorithms allows designers to calculate the requisite parameters for construction of cable-supported bridges. The program accepts a wide range of treadway parameter inputs. A result is executed in less than a minute on a computer having a clock rate of 100 MHz. Treadways with a large incline take significantly longer to calculate. This is because the required number of iterations in the root extractions increases substantially. Entry of several variations on spans, loads, and treadway weight allows creation of lookup tables (e.g., TABLE 1) for field use when a notebook computer is not available.

These calculations have been used to design and construct canopy access systems around the world, ranging from temperate to tropical forests. As canopy research expands and matures, the need for such safe and easy access will only increase. An ever-growing list of such facilities can be found at http://www.canopyaccess.com/.

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APPENDIX

The following equations were used in the mathematical derivations:

$$\cosh(z) = (e^{z} + e^{-z})/2$$

$$\sinh(z) = (e^{z} - e^{-z})/2$$

$$1 + \sinh^2(z) = \cosh^2(z)$$

The derivative with respect to z of cosh (z) is sinh (z). The arc length, $L(x)$, of the catenary from zero to x in equation 3 comes from a straightforward line integration of $(dx^2 + dy^2)^{1/2}$.