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An Evaluation of Nine Dimensionless Fetch-Limited Wind-Wave Interaction Formulas for the Northeast Gulf of Mexico

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ABSTRACT

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In the northeast Gulf of Mexico, the wind blows from 20° to 130° during approximately 40% of the year. Thus the waves generated by these winds are fetch-limited. A set of nine nondimensional formulas relating the significant wave height, dominant wave period at the spectral peak, and the fetch parameters are incorporated in this evaluation. It is found that the formulation by Dobson *et al.* performs the best amongst these nine equations. An example for the wind-stress drag coefficient determination is also provided based on this evaluation. The formulas suggested by the WAMDI Group are explained by these wind-wave interaction characteristics.

ADDITIONAL INDEX WORDS: *Significant wave height, wave period, wave age, friction velocity, drag coefficient.*

INTRODUCTION

According to the *Shore Protection Manual* (see USACE, 1984, p. 1–5 and p. 2–1), the motions of the sea which contribute to the beach and nearshore physical system include waves, tides, currents, storm surges, and tsunamis. Wind waves are by far the largest contributors of energy from the sea to the beach and nearshore physical system. Therefore, an adequate understanding of the fundamental physical processes in surface wave generation and propagation must precede any attempt to comprehend complex water motion in the nearshore areas of large bodies of water.

In certain coastal seas, the wind waves are controlled more by the overwater distance (fetch) and less by the duration of the wind. The northeastern Gulf of Mexico is such a region (see Figure 1). After the passage of atmospheric cold fronts, easterly winds usually prevail, blowing from western Florida towards the north-central Gulf. In this northeastern Gulf region (see Figure 1), approximately 30% of the time or 110 days in a given year, easterly winds prevail (see Table 1). Here we define easterly as the wind direction range from 50° to 130° from the north. Since these winds are synoptically induced, the fetch rather than the duration is the first limiting factor for the deepwater wave generation.

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Under fetch-limited conditions, there exists nine pertinent equations relating the wind-wave interaction from the open literature. Our purpose is to evaluate the performance of each of these equations as applied to the northeastern Gulf, where the wind blows from the nearby land toward our study area during 40% of the year, if one includes northerly directions from 20° to 40°.

THE FORMULAS

In order to study the wind and wave interaction, the nondimensional parameterization of the dominant wave period at the spectral peak, T_p, is given by (gT_p/U_{10}), where g is the gravitational acceleration (= 9.8 m s^{-2}), and U_{10} is the wind speed in m s^{-1} at the 10 m altitude above the mean sea surface (see, *e.g.,* USACE, 1984, pp. 3–44 to 3–50). The nondimensional wave height is given by (gH_s/U_{10}^2) where H_s is the significant wave height. The third parameter is the nondimensional fetch given by (gF/U_{10}^2) where F stands for the fetch in meters. The relationships amongst these dimensionless parameters are listed in Tables 2 and 3.

Table 1. *Annual percentage frequency of the wind direction (in tens of degrees) over the northeast Gulf of Mexico (from NDBC, 1990).*

Station	35-01	02-04	05-07	08-10	11-13	14-16	17-19	20-22	23-25	26-28	29-31	32-34
42007	7.2	8.4	9.2	9.7	12.5	11.0	9.2	9.0	7.3	6.2	5.0	5.2
42009	9.8	8.0	7.3	9.4	13.2	11.8	6.6	5.1	6.0	8.1	7.2	7.5

Easterly Winds ≃ 30% or 110 Days In A Year

Note: The period of record for station 42007 was from 1981 through 1988 and for 42009 from 1980 through 1986 (see NDBC, 1990).

Table 2. *Nine relationships between dimensionless wave period and fetch parameter.*

Reference	Formula	Eq. (#)
Hasselmann et al. (1973) JONSWAP with both field and lab data	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{3.50}\left(\frac{gF}{U_{10}^2}\right)^{0.33}$	1
Davidan (1980) JONSWAP without lab data	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{2.55}\left(\frac{gF}{U_{10}^2}\right)^{0.28}$	2
Kahma (1981)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{3.18}\left(\frac{gF}{U_{10}^2}\right)^{0.33}$	3
Donelan et al. (1985)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{1.85}\left(\frac{gF}{U_{10}^2}\right)^{0.23}$	4
Dobson et al. (1989)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{1.7}\left(\frac{gF}{U_{10}^2}\right)^{0.24}$	5
Wen et al. (1989)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{1.66}\left(\frac{gF}{U_{10}^2}\right)^{0.23}$	6
Ewans and Kibblewhite (1990)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{2.98}\left(\frac{gF}{U_{10}^2}\right)^{0.30}$	7
Babanin and Soloviev (1998)	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{2.41}\left(\frac{gF}{U_{10}^2}\right)^{0.275}$	8
Zakharov and Zaslavskii (1983) theoretical	$\left(\frac{gT_p}{U_{10}}\right) = \frac{1}{1.46}\left(\frac{gF}{U_{10}^2}\right)^{0.21}$	9

Table 3. *Nine relationships between dimensionless significant wave height and wave period.*

Reference	Formula	Eq. (#)
Babanin and Soloviev (1998)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.01152\left(\frac{gT_p}{U_{10}}\right)^{1.505}$	10
Hasselmann et al. (1976)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.00903\left(\frac{gT_p}{U_{10}}\right)^{1.667}$	11
Davidan (1980)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.01046\left(\frac{gT_p}{U_{10}}\right)^{1.47}$	12
Kahma (1981)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.01362\left(\frac{gT_p}{U_{10}}\right)^{1.50}$	13
Donelan et al. (1985)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.00958\left(\frac{gT_p}{U_{10}}\right)^{1.65}$	14
Dobson et al. (1989)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.00897\left(\frac{gT_p}{U_{10}}\right)^{1.65}$	15
Wen et al. (1989)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.01109\left(\frac{gT_p}{U_{10}}\right)^{1.515}$	16
Ewans and Kibblewhite (1990)	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.00998\left(\frac{gT_p}{U_{10}}\right)^{1.455}$	17
Zakharov and Zaslavskii (1983) theoretical	$\left(\frac{gH_s}{U_{10}^2}\right) = 0.01339\left(\frac{gT_p}{U_{10}}\right)^{1.335}$	18

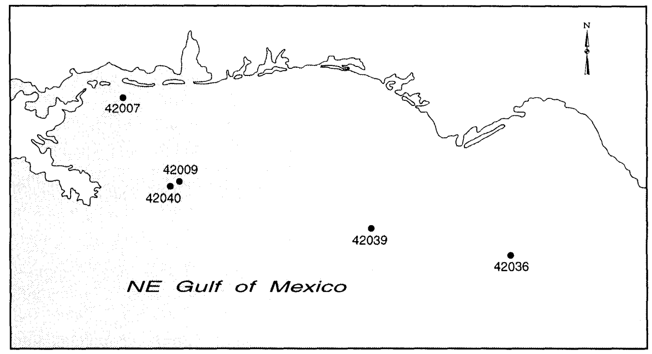

Figure 1. NDBC stations in the northeast Gulf of Mexico used in this study.

THE EVALUATION

On November 15 and 16, 1996, the northeast Gulf of Mexico experienced persistent easterly winds for at least two days as shown in Figures 2 and 3. Environmental data recorded hourly during these days at three buoy locations (Figure 2) are employed for this evaluation (48 samples). Because the anemometer height at the buoys is 5 m, wind speeds at 10 m (U_{10}) were computed using the power law relationship with P = 0.1 (see, *e.g.*, HSU *et al.*, 1994). The results for the relationship between the dimensionless fetch and wave period are summarized in Table 4 based on Table 2. Only buoy #42036 is included here since it reports the directional wave spectra. The average difference between wind and wave direction for this dataset was approximately 5°, with a maximum of 18°. It can be seen that Eq. (5) as formulated by DOBSON *et al.* (1989) worked the best in our study area. The

Table 4. *An evaluation of nine wind-wave interaction formulas based on hourly data at buoy 42036 for the northeast Gulf of Mexico (dimensionless fetch case).*

Equation	gF/U_{10}^2	gT_p/U_{10} Predicted	gT_p/U_{10} Observed	(P − O)/O % Difference	Ranking
1	9068	5.78	5.28	9.47	6
2	9068	5.03	5.28	−4.73	4
3	9068	6.36	5.28	20.45	9
4	9068	4.40	5.28	−16.67	8
5	9068	5.24	5.28	−0.76	1
6	9068	4.90	5.28	−7.20	5
7	9068	5.16	5.28	−2.27	2
8	9068	5.09	5.28	−3.60	3
9	9068	4.64	5.28	−12.12	7
Equation	gT_p/U_{10}	gH_s/U_{10}^2 Predicted	gH_s/U_{10}^2 Observed	(P − O)/O % Difference	
10	5.28	0.141	0.149	−5.37	
11	5.28	0.145	0.149	−2.68	
12	5.28	0.121	0.149	−18.79	
13	5.28	0.165	0.149	10.74	
14	5.28	0.149	0.149	0.0	
15	5.28	0.140	0.149	−6.04	
16	5.28	0.138	0.149	−7.38	
17	5.28	0.112	0.149	−24.83	
18	5.28	0.123	0.149	−17.45	

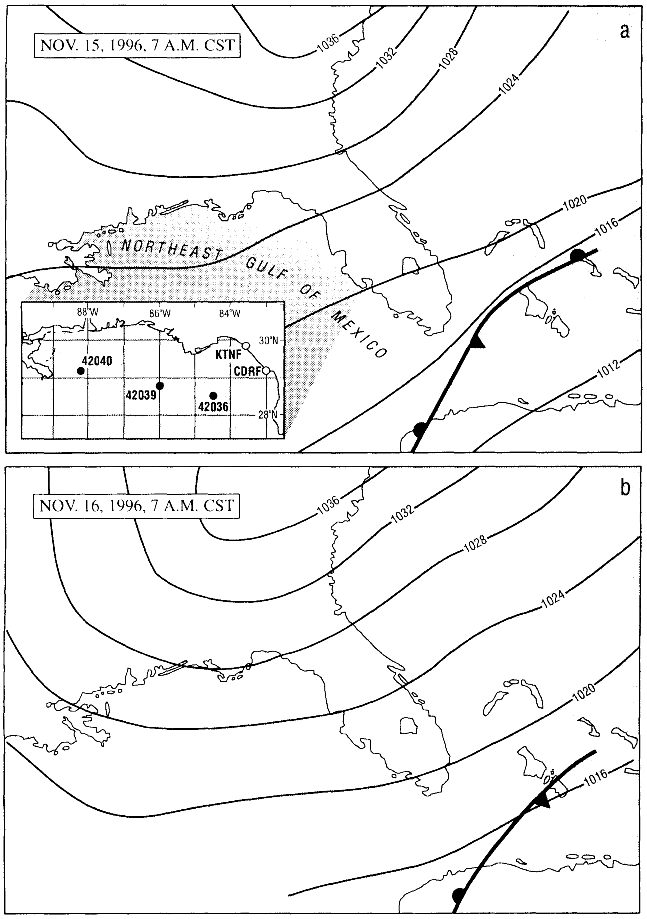

Figure 2. Simplified surface weather maps for November 15 (top) and 16 (bottom), 1996 at 7 A.M. CST and the three NDBC buoys used in this evaluation.

findings for the relationship between dimensionless significant wave height and dominant wave period based on Table 3 is provided in Table 5. Again, the formulation by DOBSON *et al.* (1989) ranked the best. Note that Tables 4 and 5 present the mean values based on the hourly data.

On the basis of these evaluations, we recommend that Eqs. (5) and (15) as formulated by DOBSON *et al.* (1989) be used operationally in the northeast Gulf of Mexico.

AN APPLICATION FOR WIND-STRESS DETERMINATION

In wind-wave interaction studies, the wind-stress input is essential (see, *e.g.*, the WAMDI GROUP, 1988). The wind stress, τ, is defined as

$$\tau = \rho u_*^2 = \rho C_d U_{10}^2 \tag{19}$$

where ρ is the air density, u_* is the friction or shear velocity, C_d is the drag coefficient, and U_{10} is the wind speed at 10 m above the sea surface.

Since the parameter u_* is an integral part of the wind-wave

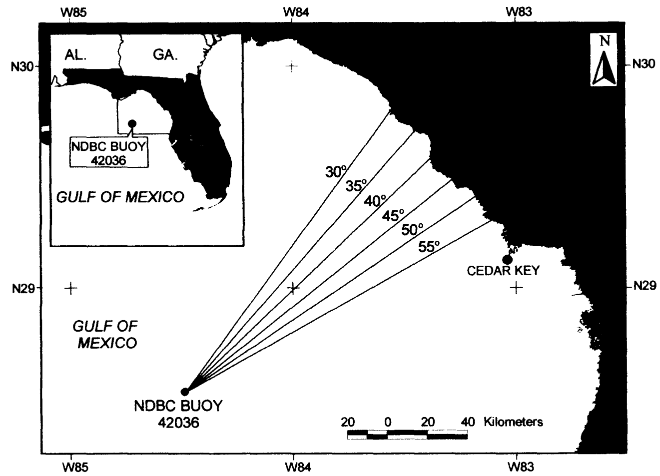

Figure 3. An example of the wind direction from 30° to 55° from the north for the measured fetch to buoy #42036 (see Fig. 1) as used in Table 4. The distances are acquired using MGE (Modular GIS Environment) software. The map of the Gulf of Mexico with Albert Equal Area projection was used. After determining the fetch directions based on true north, distances from buoy 42036 to the closest shoreline were calculated.

and other air-sea interaction studies (see, *e.g.*, HSU, 1988), we will apply our results to the u_* determination as follows:

From Eq. (19)

$$u_* = U_{10}\sqrt{C_d} \tag{20}$$

From the WAMDI GROUP (1988)

$$C_d = \begin{cases} 1.2875 \times 10^{-3}, & U_{10} < 7.5 \text{ m s}^{-1} \\ (0.8 + 0.065U_{10}) \times 10^{-3}, & U_{10} \geq 7.5 \text{ m s}^{-1} \end{cases} \tag{21}$$

This choice of expression for the drag coefficient was found to be crucial in resolving some remarkable differences between wave growth datasets collected in the North Sea. No further corrections, for example for atmospheric stability, were applied.

On the other hand, from the Wind-Wave Interaction method (HSU, 1995)

Table 5. *An evaluation of nine wind-wave interaction formulas based on hourly data for the northeast Gulf of Mexico (dimensionless wave period vs. significant wave height).*

Equation	#42036	#42039	#42040	Mean	Ranking
10	−5.37	−2.37	3.39	−1.45	2
11	−2.68	1.78	9.04	2.71	3
12	−18.79	−16.57	−11.86	−15.74	8
13	10.74	14.79	20.90	15.48	7
14	0.0	5.33	12.43	5.92	5
15	−6.04	−1.78	5.08	−0.91	1
16	−7.38	−4.14	1.13	−3.46	4
17	−24.83	−22.49	−18.08	−21.80	9
18	−17.45	−15.98	−11.86	−15.10	6

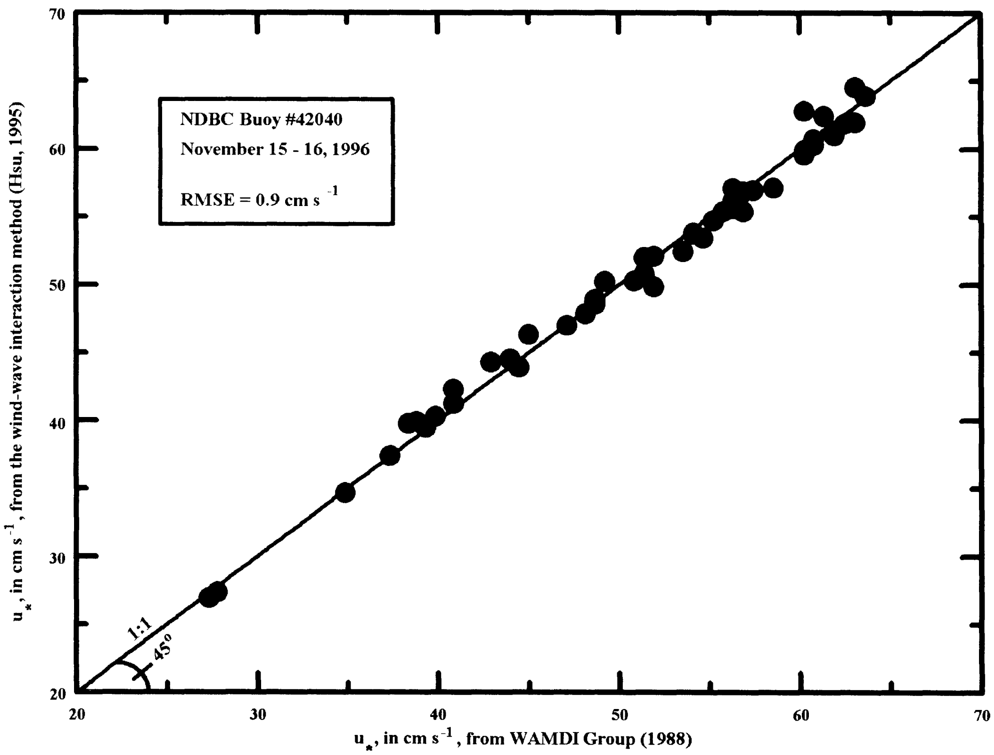

Figure 4. A comparison of the shear velocity u_* based on Eqs. (21) and (22) using buoy #42040 during November 15 and 16, 1996 in our study area (see Fig. 1).

$$u_* = \frac{0.4U_{10}}{11.0 - \ln\left(\frac{H_s}{\left(\frac{C_p}{U_{10}}\right)^{2.6}}\right)} \tag{22}$$

where $C_p = gT_p/2\pi$, is the phase speed of the waves at the spectral peak. Note that the parameter C_p/U_{10} is defined as the wave age.

Figure 4 shows the results of our comparison between Eqs. (21) and (22). Since the root mean square error is only 0.9 cm s^{-1} for the data range between 20 and 70 cm s^{-1}, either Eq. (21) or (22) can be substituted into Eq. (20). Since Eq. (21) is easier to use it is recommended for operational computations, although Eq. (22) can be used to explain Eq. (21) based on wind-wave interaction characteristics.

CONCLUSIONS

On the basis of the foregoing evaluations we conclude that

(1) In the northeast Gulf of Mexico approximately 40% of the time in a year, the wind blows from 20° to 130° from the north and east (land to sea), thus the wind waves are generated in fetch-limited conditions;

(2) Out of nine nondimensional wind-wave interaction formulations the one by DOBSON *et al.* (1989) ranks as the best in our area during the fetch-limited conditions; and

(3) For further wind-wave and air-sea interaction studies the drag coefficient as formulated by the WAMDI Group has been explained by the wind-wave interaction and is recommended for use in the northeast Gulf of Mexico.

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