Journal of Coastal Research	16	2	379–384	Royal Palm Beach, Florida	Spring 2000

Partial Standing Waves on a Steep Slope

Nobuhisa Kobayashi†, Giuseppe R. Tomasicchio‡ and Bruno Brunone‡

†Center for Applied Coastal Research
University of Delaware
Newark, DE 19716, U.S.A.

‡Institute of Hydraulics
University of Perugia
Guranti 1/4, 06125 Perugia, Italy

ABSTRACT

KOBAYASHI, N.; TOMASICCHIO, G.R., and BRUNONE, B., 2000 Partial standing waves on a steep slope. *Journal of Coastal Research*, 16(2), 379–384. Royal Palm Beach (Florida), ISSN 0749-0208.

Measured free surface elevations and horizontal velocities of non-breaking regular waves on a 1:2 rough permeable slope are analyzed to examine the cross-shore variations of the incident and reflected waves on the steep slope. The co-located measurements of the free surface elevation and horizontal velocities at a number of elevations are used to separate the incident and reflected waves locally. The estimated partial standing waves are then shown to describe the local free surface elevation and horizontal velocity reasonably well. The incident wave energy flux is found to be approximately constant along the 1:2 slope, whereas the data and linear theory are not accurate enough to detect the cross-shore variation of the relative small wave reflection coefficient. The measured mean horizontal velocities are shown to be in qualitative agreement with the return current estimated from the separated incident and reflected waves.

ADDITIONAL INDEX WORDS: *Reflection, standing waves, coastal structures, velocity profile, energy flux, energy dissipation, return current.*

INTRODUCTION

Partial standing waves occur on a horizontal bottom in front of an inclined coastal structure (HUGHES and FOWLER, 1995). Partial standing waves may also occur on the steep slope of a coastal structure located in relatively deep water under moderate wave conditions. Such partial standing waves have not been investigated previously for lack of detailed measurements on nonbreaking waves on the steep rough slope except for regular wave data obtained by BRUNONE and TOMASICCHIO (1996, 1997) who analyzed the vertical distribution of the measured horizontal velocity using its variance. To explain the measured velocity distribution in a simple manner, their data and one additional test data are reanalyzed here, assuming that the linear wave theory developed for a horizontal bottom is approximately valid locally even on the steep slope. The assumption allows one to estimate the local incident and reflected waves using the co-located measurements of the free surface elevation and horizontal velocity (HUGHES, 1993). The repeated co-located measurements along the slope are then used to examine the cross-shore variations of the incident and reflected waves where BAQUERIZO *et al.* (1997, 1998) already examined the cross-shore variations of wave reflection from beaches.

In the following, the method used to estimate the incident and reflected waves from the measured free surface elevation and horizontal velocities on a 1:2 rough permeable slope is explained concisely. The energy equation associated with the estimated incident and reflected waves is then examined to assess the degree of importance of the reflected wave energy flux and the energy dissipation rate due to bottom friction. Finally, the vertical profile of the measured mean horizontal velocity on the steep slope is presented for the first time and compared with the return current based on linear wave theory.

98191 received 29 June 1998; accepted in revision 7 June 1999.

SEPARATION OF INCIDENT AND REFLECTED WAVES

BRUNONE and TOMASICCHIO (1996, 1997) conducted five regular wave tests on a 1:2 rough permeable slope with an impermeable core in a wave flume that was 35 m long, 0.75 m wide, and 1.5 m deep. The experiment setup was presented in their papers. The nominal diameter of the armor stone was $D_n = 2.7$ cm. The water depth below the still water level (SWL) on the horizontal bottom was 50 cm. The incident waves generated by a piston type wavemaker surged on the 1:2 slope with little or no wave breaking and caused no damage on the armor layer. The five tests corresponded to weakly-nonlinear waves in relatively deep water in the absence of reflected waves (BRUNONE and TOMASICCHIO, 1996). One additional test conducted in the same manner is included in the following data analysis. The time series of the free surface elevation η above SWL was measured by standard conductivity-type gauges at $x = 0, 15, 35, 40, 50$ and 60 cm for each test where x is the horizontal coordinate taken to be positive landward with $x = 0$ at the toe of the slope. The still water depth h is given by $h = (50 - x/2)$ cm. The vertical coordinate z is taken to be positive upward with $z = 0$ at SWL. The horizontal velocities u at each location of the free surface measurement were measured with a single micropropeller at different elevations with a vertical interval of about 3.5 cm below wave trough level and sufficiently above the bottom boundary layer on the armor layer. The measured horizontal velocities presented by BRUNONE and TOMASICCHIO (1996, 1997) did not indicate the effect of the sloping bottom on the vertical variations of the horizontal velocities. The sampling rate of these measurements was 20 Hz.

The measured time series of η and u for the duration of 10 wave periods after the establishment of quasi-periodic wave motions are used to calculate the mean values, $\bar{\eta}$ and $\bar{u}$, where the overbar denotes time averaging. In these tests, the mean elevation $\bar{\eta}$ is on the order of 1 mm and much smaller than the oscillatory component $(\eta - \bar{\eta})$ which is on the order of 3 cm. The measured mean elevation $\bar{\eta}$ is not accurate enough to detect the cross-shore variation of small wave set-down or set-up. On the other hand, the measured mean horizontal velocity $\bar{u}$ is negative (seaward) and on the order of 1 cm/s in comparison to the oscillatory component $(u - \bar{u})$ which is on the order of 10 cm/s. The measured mean velocity $\bar{u}$ will be presented after the analysis of the oscillatory components $(\eta - \bar{\eta})$ and $(u - \bar{u})$ which are simply denoted by η and u in the following.

The oscillatory components η and u are assumed to be the sum of incident and reflected wave components

$$\eta(t, x) = \eta_i(t, x) + \eta_r(t, x);$$

$$u(t, x, z) = u_i(t, x, z) + u_r(t, x, z) \quad (1)$$

where the subscripts i and r indicate the incident and reflected waves, respectively, and t is time with $t = 0$ at the start of 10 wave periods. The free surface oscillations of η_i and η_r are assumed to be sinusoidal

$$\eta_i(t, x) = a_i(x)\cos[\phi_i(x) - \sigma t];$$

$$\eta_r(t, x) = a_r(x)\cos[\phi_r(x) + \sigma t]; \quad (2)$$

with

$$\frac{d\phi_i}{dx} = k(x); \qquad \frac{d\phi_r}{dx} = k(x) \quad (3)$$

$$\sigma^2 = gk(x)\tanh[k(x)h(x)] \quad (4)$$

where a = local amplitude; ϕ = local phase function; σ = angular frequency given by $\sigma = 2\pi/T$ with T = wave period; k = local wave number; g = gravitational acceleration; and h = local still water depth. The linear dispersion relation given by (4) is assumed to be valid locally. The parameters a_i, a_r, ϕ_i, ϕ_r and k are allowed to vary spatially. Using linear wave theory (DEAN and DALRYMPLE, 1984), the horizontal velocities u_i and u_r are expressed as

$$u_i(t, x, z) = F(x, z)\eta_i(t, x);$$

$$u_r(t, x, z) = -F(x, z)\eta_r(t, x) \quad (5)$$

with

$$F(x, z) = \frac{gk(x)}{\sigma}\frac{\cosh\{k(x)[h(x) + z]\}}{\cosh[k(x)h(x)]} \quad (6)$$

To estimate the local values of a_i, a_r, ϕ_i and ϕ_r, the measured time series of η and u are fitted in the following forms:

$$\eta(t, x) = P(x)\cos(\sigma t) + Q(x)\sin(\sigma t) \quad (7)$$

$$u(t, x, z) = C(x, z)\cos(\sigma t) + D(x, z)\sin(\sigma t) \quad (8)$$

where a_i, a_r, ϕ_i and ϕ_r can be expressed in terms of P, Q, C and D (HUGHES, 1993). The values of P and Q at given x are obtained by minimizing the square of the difference between the measured and fitted values of η over the duration of 10 waves. The error e_η of this fit is estimated as the ratio between the standard deviation of the difference based on (7) and the standard deviation of the measured η. Likewise, the values of C and D at given x and z are obtained by minimizing the square of the difference between the measured and fitted values of u over the duration of 10 waves. The error e_u of the velocity fit is estimated as the ratio between the standard deviation of the difference based on (8) and that of the measured u.

Table 1. *Incident and reflected wave amplitudes a_i and a_r for screened data*

Test	T (s)	x (cm)	h (cm)	L (cm)	N_u	a_i (cm)	a_r (cm)
1	0.80	40	30	96	2	1.97	0.18
		50	25	93	3	1.99	0.46
		60	20	89	4	1.88	0.50
2	1.70	15	42.5	313	10	2.59	0.84
		30	35	289	7	2.73	0.61
		40	30	271	6	2.88	0.84
		50	25	251	4	3.05	1.21
3	1.25	0	50	218	12	2.15	0.88
		15	42.5	209	10	2.10	0.90
		50	25	175	5	2.18	0.56
4	1.00	0	50	151	4	2.87	0.44
		15	42.5	148	8	2.83	0.29
		30	35	142	7	2.87	0.75
		40	30	137	7	2.76	0.85
		50	25	130	5	2.72	0.79
		60	20	121	2	2.76	0.55

The measured free surface oscillations for the six tests can be expressed in the form of (7) with the error e_η less than 10%. However, some of the measured horizontal velocities deviated significantly from the assumed form of (8) perhaps because of some irregularities in the measured velocities. Only the measured horizontal velocities with the error e_u less than 20% are analyzed hereafter where the error of 20% might be regarded as an acceptable limit of linear wave theory for practical applications.

Table 1 summarizes the screened data sets for four tests where $L = (2\pi/k)$ is the local wavelength and N_u is the number of the horizontal velocity measurements at given x which satisfy the requirement of e_u less than 20%. Tests 1, 2 and 3 correspond to the wave conditions 1, 4 and 5 in BRUNONE and TOMASICCHIO (1997), whereas test 4 is the new test added here. Only the location x with $N_u \geq 2$ is considered so that the values of a_i, a_r, ϕ_i and ϕ_r can be estimated N_u times using the values of C and D at N_u elevations. The estimated values of a_i, a_r, ϕ_i and ϕ_r using the N_u pairs of C and D are found to be fairly consistent with deviations less than 20%. Table 1 lists the average values of a_i and a_r at each location x. The average values of ϕ_i and ϕ_r obtained at each location x are checked using the phase relations of ϕ_i and ϕ_r given by (3) which is approximated by a finite difference. The estimated incident waves satisfy this phase relation within the error less than 20%. However, the estimated reflected waves do not always satisfy the phase relation within this error range and may not be very reliable. It is noted that the measurements in these tests were limited to the region where the amplitudes a_i and a_r are much smaller than the local depth h as listed in Table

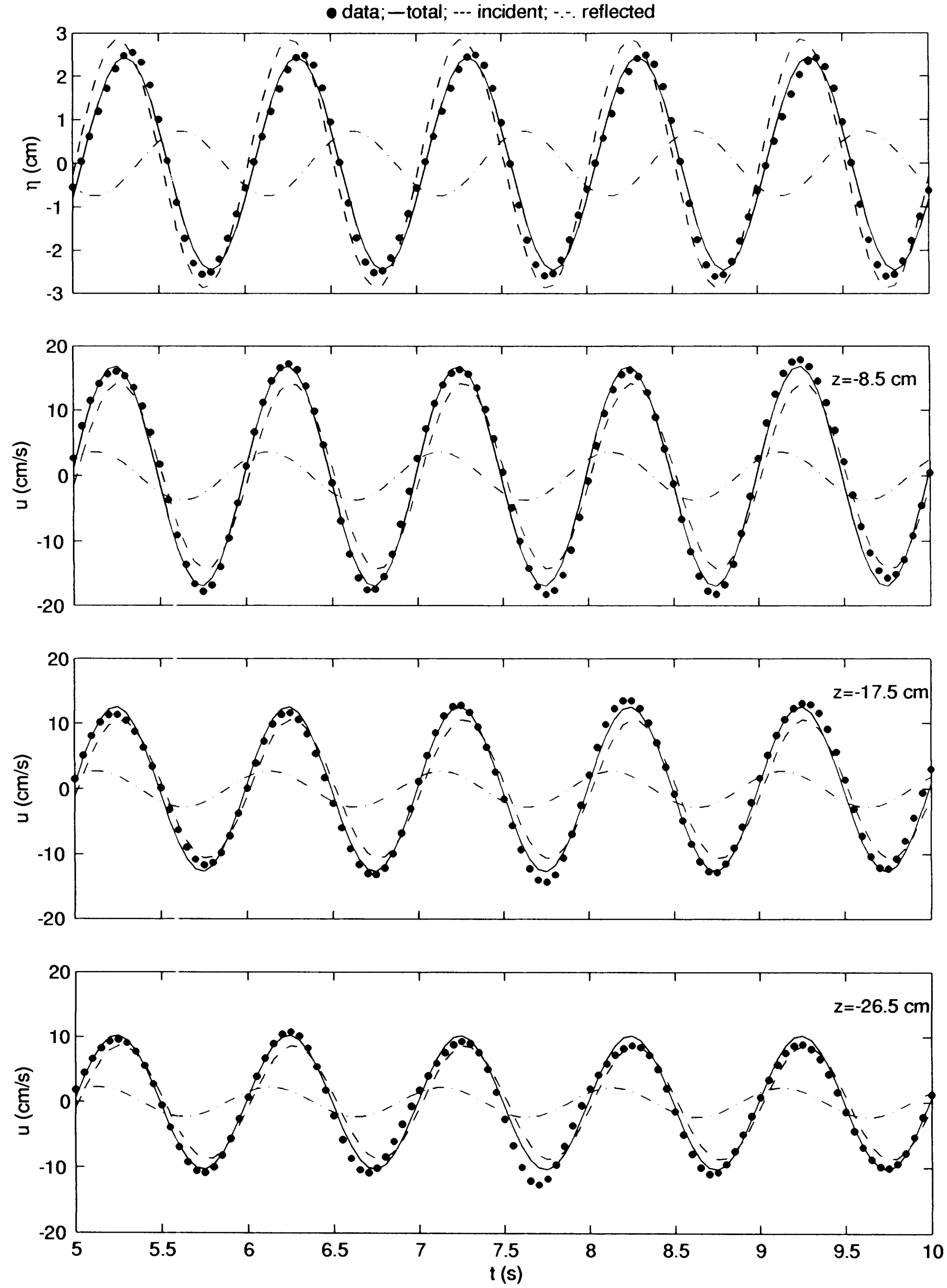

Figure 1. Temporal variations of free surface elevation η and horizontal velocity u at elevation $z = -8.5$, -17.5, and -26.5 cm for test 4 and $x = 30$ cm.

1. Furthermore, the local wavelength L does not change significantly with respect to x in the region of the measurements.

To examine the degree of the accuracy of this linear analysis, Figure 1 shows the measured and fitted time series for five wave periods of $\eta = (\eta_i + \eta_r)$ and those of $u = (u_i + u_r)$ at $z = -8.5$, -17.5, and -26.5 cm at the location $x = 30$ cm with $h = 35$ cm for test 4 where the average values of a_i and a_r listed in Table 1 are used in Figure 1 and the subsequent figures. In Figure 1, $e_\eta = 0.12$, whereas $e_u = 0.11$, 0.12 and 0.14 at $z = -8.5$, -17.5 and -26.5 cm, respectively. These errors are typical for the screened data listed in Table 1. The degree of the fit for the horizontal velocity u remains approximately the same at different elevations, implying that the vertical variation of the horizontal velocity u can be described

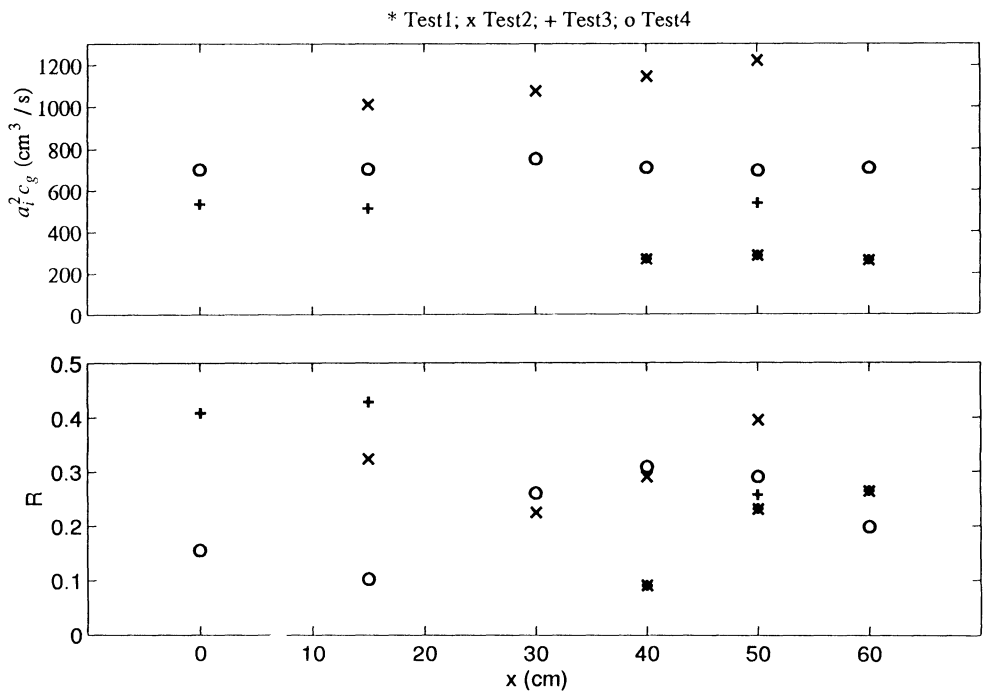

Figure 2. Cross-shore variations of $a_i^2\ c_g$ and $R\ =\ a_r/a_i$ for four tests.

by (6). The measured time series of η and u are not completely periodic and the errors e_η and e_u are partly caused by these irregularities as well as by weak nonlinearity. Figure 1 shows that the reflected wave components of the free surface elevation and horizontal velocity are not negligible on the steep slope.

WAVE ENERGY EQUATION

The estimated incident and reflected waves are used to examine the wave energy balance on the steep rough slope. The wave energy equation including the rate of energy dissipation due to bottom friction estimated using the formula by JONSSON (1966) may simply be expressed as

$$\frac{d}{dx}(a_i^2 c_g - a_r^2 c_g) = -\frac{4 f_w u_b^3}{3\pi g} \tag{9}$$

where c_g = group velocity; f_w = wave friction factor; and u_b = amplitude of the linear wave horizontal velocity u at the bottom $z = -h$ which can be found using (5) and (6) including both incident and reflected waves. For these small-scale tests, u_b is on the order of 10 cm/s and the corresponding water particle amplitude, $A_b = u_b/\sigma$, is on the order of the stone diameter D_n = 2.7 cm. The friction factor f_w under these conditions is uncertain but may be assumed to be on the order of 0.3 in light of limited available data (MADSEN and WHITE, 1976; CORNETT and MANSARD, 1994).

Figure 2 shows the cross-shore variations of $a_i^2 c_g$ and $R = a_r/a_i$ for the screened data listed in Table 1 where R is the local reflection coefficient. The ratio, $(a_r^2 c_g/a_i^2\ c_g) = R^2$, is relatively small for these tests. The estimated values of the right hand side of (9) with f_w = 0.3 are on the order of 0.1 cm^2/s. As a result, the rate of energy dissipation due to bottom friction in the region $0 < x < 60$ cm may be negligible. Under the conditions of the negligible energy dissipation and $R^2 \ll 1$, (9) predicts that $a_i^2 c_g$ is approximately constant in agreement with the data for each test shown in Figure 2. As a first approximation, the incident linear waves shoal on the steep slope. As for the reflected waves, the data and the adopted linear analysis are not accurate enough to examine whether the reflected waves shoal inversely (BAQUERIZO *et al.*, 1997) and whether the incident waves are reflected from the steep slope in the region $0 < x < 60$ cm (BAQUERIZO *et al.*, 1998). It should be noted that the values of the reflection coefficient R shown in Figure 2 are in qualitative agreement with the empirical formulas proposed by SEELIG and AHRENS (1995), HUGHES and FOWLER (1995), and DAVIDSON *et al.*, (1996). Most of the incident wave energy flux represented by $a_i^2 c_g$ in Figure 2 must have been dissipated in the shallower region near the shoreline.

RETURN CURRENT

Figure 3 shows the vertical profile of the measured mean horizontal velocity $\bar{u}$ for all the screened data listed in Table 1. The magnitude of $\bar{u}$ is compared with the return current $\bar{U}$ obtained using linear wave theory in the following. The time-averaged volume flux $\bar{q}$ in the direction of the incident wave propagation is given by (DEAN and DALRYMPLE, 1984)

$$\bar{q} = \overline{\int_{-h}^{\eta} u\, dz} \simeq \overline{u(t, x, z = 0)\eta(t, x)} \tag{10}$$

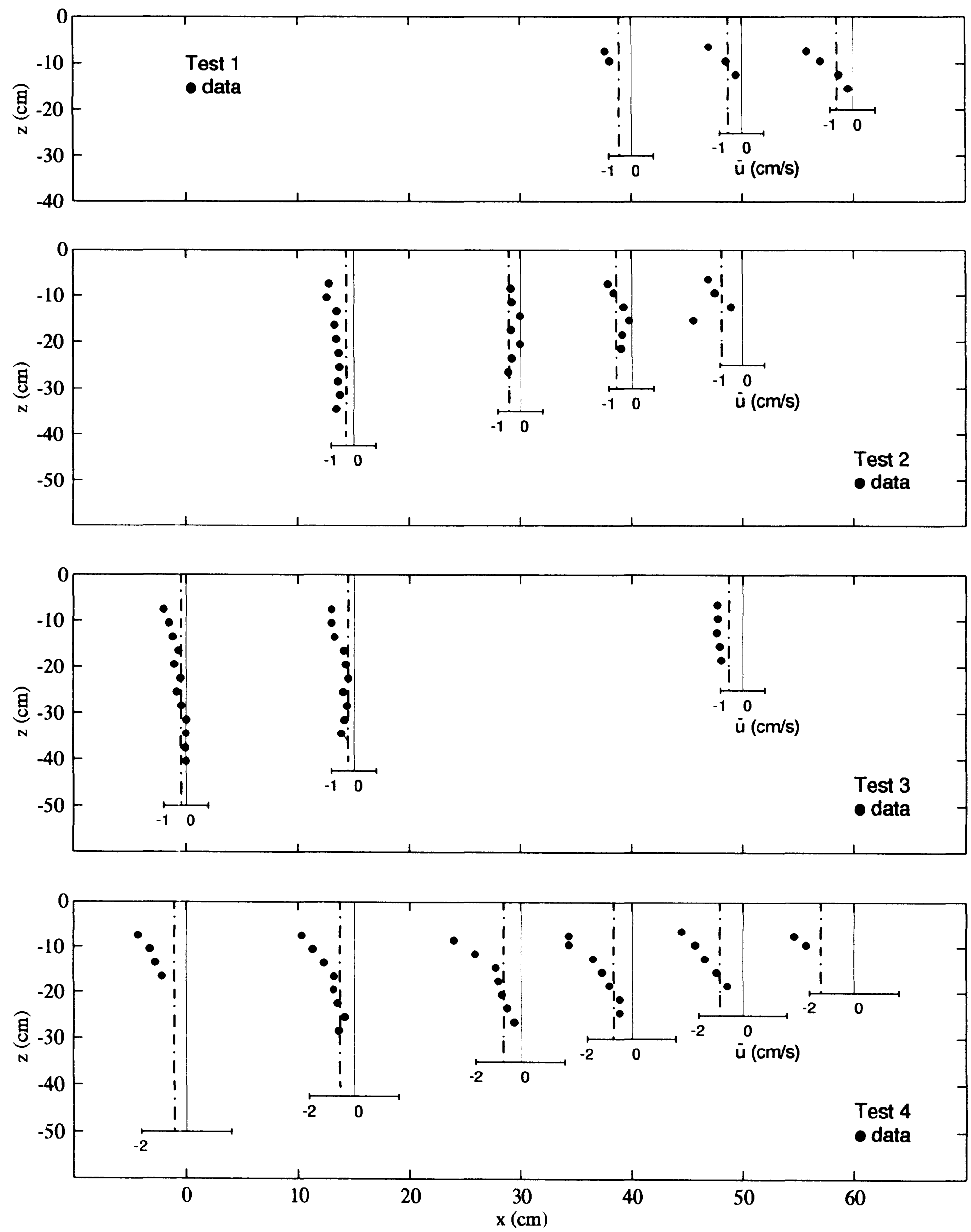

Figure 3. Vertical and cross-shore variations of measured mean horizontal velocity $\bar{u}$ in comparison with return current based on linear wave theory for four tests.

to the second-order accuracy. Substitution of (1) with (2) and (5) into (10) yields

$$\bar{q} \simeq \frac{g}{2c}(a_i^2 - a_r^2) \tag{11}$$

where $c = L/T$ is the local celerity. The return current $\bar{U}$ given by $\bar{U} = -\bar{q}/h$ is necessary to satisfy no flux condition into the impermeable structure. The return current $\bar{U}$ does not change vertically for potential wave theory. The constant value of $\bar{U}$ at given x for each test is calculated using the values of a_i and a_r listed in Table 1 and plotted in Figure 3. Eq. (11) implies that the reflected waves reduce the magnitude of the return current and can not be neglected unless $(a_r/a_i)^2 \ll 1$ (KENNEDY *et al.*, 1997).

Figure 3 indicates that the return current $\bar{U}$ based on linear wave theory represents the measured mean horizontal velocity $\bar{u}$ at least qualitatively. The measured values of $|\bar{u}|$ tend to increase upward below wave trough level, especially for test 4. This trend is similar to the undertow profile outside the surf zone on a beach (COX and KOBAYASHI, 1997). However, the measured values of $\bar{u}$ tend to fluctuate vertically and may not be very accurate.

CONCLUSIONS

The free surface elevations and horizontal velocities measured in the relatively deep water on a 1:2 rough permeable slope are analyzed using linear wave theory. The measured oscillatory components of the free surface elevations and horizontal velocities are shown to be consistent with linear partial standing waves. In the region of the measurements, the energy dissipation due to bottom friction is estimated to be negligible, and the incident waves shoal on the steep slope as a first approximation. The data and linear theory are not accurate enough to resolve the cross-shore variation of relatively small reflected waves. The measured mean horizontal velocities are represented reasonably by the return current based on linear wave theory. These results may have been expected but have never been presented for lack of data on a steep slope. On the other hand, the present linear analysis may not hold in the shallow water near the shoreline where most energy dissipation and wave reflection appear to occur. Non-linear shallow water theory including the effects of bottom friction and permeability (KOBAYASHI and WURJANTO, 1990) will be necessary to analyze the wave motion near the shoreline. The comparison of the present data with the predictive model of BAQUERIZO *et al.* (1998) for wave reflection from beaches may be worthwhile but is not attempted here.

LITERATURE CITED

BAQUERIZO, A.; LOSADA, M.A., and SMITH, J.M., 1998. Wave reflection from beaches: A predictive model. *Journal of Coastal Research,* 14(1), 291–298.

BAQUERIZO, A.; LOSADA, M.A.; SMITH, J.M., and KOBAYASHI, N., 1997. Cross-shore variation of wave reflection from beaches. *Journal of Waterway, Port, Coastal and Ocean Engineering* (ASCE), 123(5), 274–279.

BRUNONE, B. and TOMASICCHIO, G.R., 1996. Wave induced velocities at a rubble mound breakwater. *Proceedings, 25th Coastal Engineering Conference* (ASCE), 2, 1569–1582.

BRUNONE, B. and TOMASICCHIO, G.R., 1997. Wave kinematics at steep slopes: Second-order model. *Journal of Waterway, Port, Coastal and Ocean Engineering* (ASCE), 123(5), 223–232.

CORNETT, A. and MANSARD, E., 1994. Wave stresses on rubble-mound armor. *Proceedings, 24th Coastal Engineering Conference* (ASCE), 1, 986–1000.

COX, D.T. and KOBAYASHI, N., 1997. A kinematic undertow model with a logarithmic boundary layer. *Journal of Waterway, Port, Coastal and Ocean Engineering* (ASCE), 123(6), 354–360.

DAVIDSON, M.A.; BIRD, P.A.D.; HUNTLEY, D.A., and BULLOCK, G.N., 1996. Prediction of wave reflection from rock structures: An integration of field & laboratory data. *Proceedings, 25th Coastal Engineering Conference* (ASCE), 2, 2077–2086.

DEAN, R.G. and DALRYMPLE, R.A., 1984. *Water Wave Mechanics for Engineers and Scientists.* Singapore World Scientific.

HUGHES, S.A., 1993. Laboratory wave reflection analysis using co-located gages. *Coastal Engineering,* 20, 223–247.

HUGHES, S.A. and FOWLER, J.E., 1995. Estimating wave-induced kinematics at sloping structures. *Journal of Waterway, Port, Coastal and Ocean Engineering* (ASCE), 121(4), 209–215.

JONSSON, I.G., 1966. Wave boundary layers and friction factors. *Proceedings, 10th Coastal Engineering Conference* (ASCE), 1, 127–148.

KENNEDY, D.L.; RATHBUN, J.R.; COX, D.T., and EDGE, B.L., 1997. Irregular wave transformation and undertow for coastal structures in a surf zone. *Proceedings Waves'97* (ASCE), 2, 1115–1128.

KOBAYASHI, N. and WURJANTO, A., 1990. Numerical model for waves on rough permeable slopes. *Journal of Coastal Research,* SI(7), 149–166.

MADSEN, O.S. and WHITE, S.M., 1976. Energy dissipation on a rough slope. *Journal of Waterway, Port, Coastal and Ocean Engineering* (ASCE), 102(1), 31–48.

SEELIG, W.N. and AHRENS, J.P., 1995. Wave reflection and energy dissipation by coastal structures. *Wave Forces on Inclined and Vertical Wall Structures,* N. Kobayashi and Z. Demirbilek, eds., (ASCE), 28–51.