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## TECHNICAL COMMUNICATION

# Dune Scarp Exhibits Bifurcation Sequence

**Douglas A. Gaffney**

U.S. Army Corps of Engineers, Philadelphia District
100 Penn Square East, Wanamaker Bldg.
Philadelphia, PA 19107, U.S.A.

**ABSTRACT**

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A natural display of bifurcation is observed and related to deterministic chaos theory. A possible scenario is proposed from which an eroding dune scarp could be described.

ADDITIONAL INDEX WORDS: *Chaos theory, coastal dunes, fractal, geomorphology.*

## INTRODUCTION

The study of chaotic dynamics has lead to the observation that simple equations can describe extremely complex patterns which resemble natural systems. Apparent random behavior can be described in many instances by deterministic mathematics. In this paper, the geometry of an eroding dune scarp appears to exhibit a bifurcation sequence predicted by deterministic chaos theory.

Bifurcation is the process of division into two branches. In the case of an equation, the period is said to double when the stable solution bifurcates. In the case of deterministic chaos, the period doubling occurs with an inherent, universal regularity as will be described in the following paragraphs.

Figure 1 is a photograph of a dune scarp at Higbee Beach, New Jersey, taken in July 1991. The eroding scarp is very steep, with approximately 12 to 15 feet of elevation showing in the photograph. The bifurcation sequence shown in Figure 2 looks strangely similar to the face of the dune scarp.

## THEORY

The bifurcation sequence is a result of repeated iterations of a nonmonotonic function, having feedback which results in a function whose graph folds back on itself (HOFSTADTER, 1981). In this case, the logistic difference equation, used by ecologists to predict populations, has been applied to generate Figure 2:

$$X_{i+1} = X_i\, r(1 - X_i) \qquad \text{eq. 1.}$$

where r is the growth parameter and X is the population. Figure 2 is a map of the iteration results of X for each r, the mathematical basis for which is described by DEVANEY (1989). In the range of $2 < r < 4$, and $0 < X_i < 1$, the equation results in a single stable solution which bifurcates at a critical value, $r_{cr}$, into two stable solutions, then four, eight, sixteen, and eventually into apparent, complete randomness.

The logistic equation has a more general application than for population growth, since it incorporates the basic tenant of feedback. The parameter r is generally known as a control parameter, as it controls the amount of feedback. Further examples of feedback can be found in natural dynamic systems such as erosion (PHILLIPS, 1992), climate, disease and evolution (GLEICK, 1987).

The bifurcation sequence can be generated using many equations other than equation 1. This finding coincides with the belief that a certain universality exists in the transition to chaos (HOFSTADTER, 1981). Mitchell Feigenbaum found

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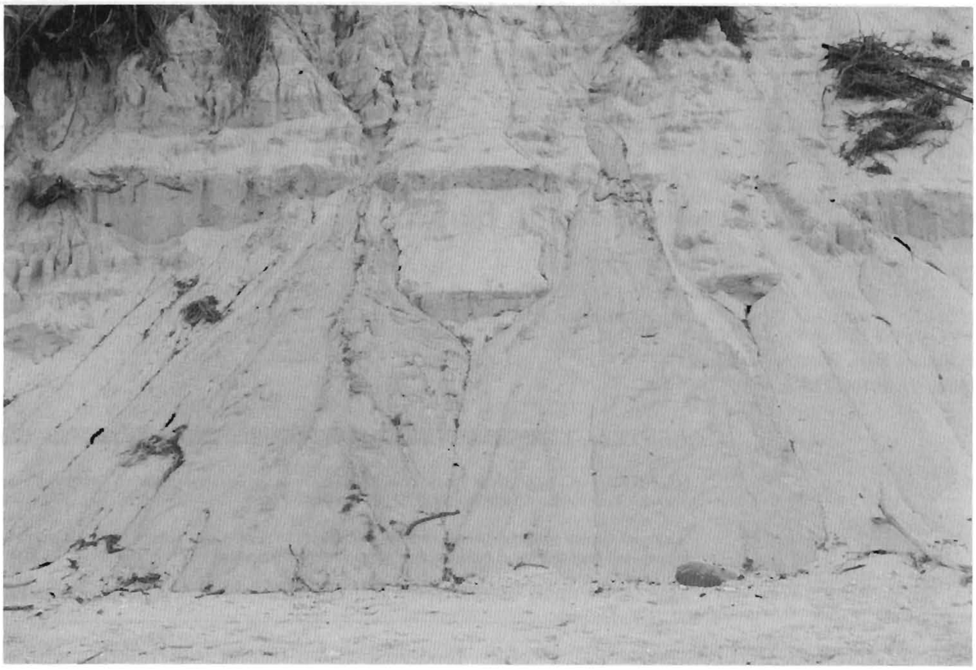

Figure 1. Dune scarp at Higbee Beach, New Jersey (July 1991).

that for all nonmonotonic functions which exhibit period doubling (GLEICK, 1987), the rate is given by the limit of n as the iterations approach infinity (INFELD and ROWLANDS, 1990):

$$\lim_{n\to\infty} \frac{\Delta r_{n+1}}{\Delta r} = 1/4.669201 \ldots \quad \text{eq. 2.}$$

This number, 4.669201 . . . , is often referred to as the universal constant. The importance of this revelation is that regardless of either the equation or the region of the map shown in Figure 2, the rate at which the bifurcations occur is uniform. Self similar bifurcations can therefore be found at many different scales within the sequence, hence the fractal character of the sequence.

Furthermore, for $r > 3.828$, the apparent chaos gives way to order. Within the bands of chaos, self similar, periodic bifurcation sequences appear, having an initial period three and a rate of doubling given by equation 2.

## DISCUSSION

The relevance of mathematical oddities such as the fractal nature of the bifurcation sequence to the real physical world is still being debated. Indeed, empirical evidence from turbulence studies cannot show more than the first few bifurcations. The control parameter, r, can represent a governing parameter of many natural processes. In the original logistic equation, r represented the fecundity of a species. Other parameters such as Reynolds number, temperature gradient, and angular velocity have been used as control parameters in past research (INFELD and ROWLANDS, 1990).

Geomorphic systems have been evaluated as well for their likelihood of exhibiting chaotic dynamics (PHILLIPS, 1992). If erosion of the dune scarp in Figure 1 were caused by a nonlinear deterministic process, then it would be conceivable to observe evidence of that process in the geometry. A scenario for the bifurcation sequence to be a model of an eroding dune scarp is proposed as follows.

Equation 1 can be applied to the erosional process, keeping cognizant of each variable's behavior and the feedback required of the iterative process. Each grain of sand falling down the face of the dune scarp can be considered to be an iteration of equation 1. The control parameter, r, can be

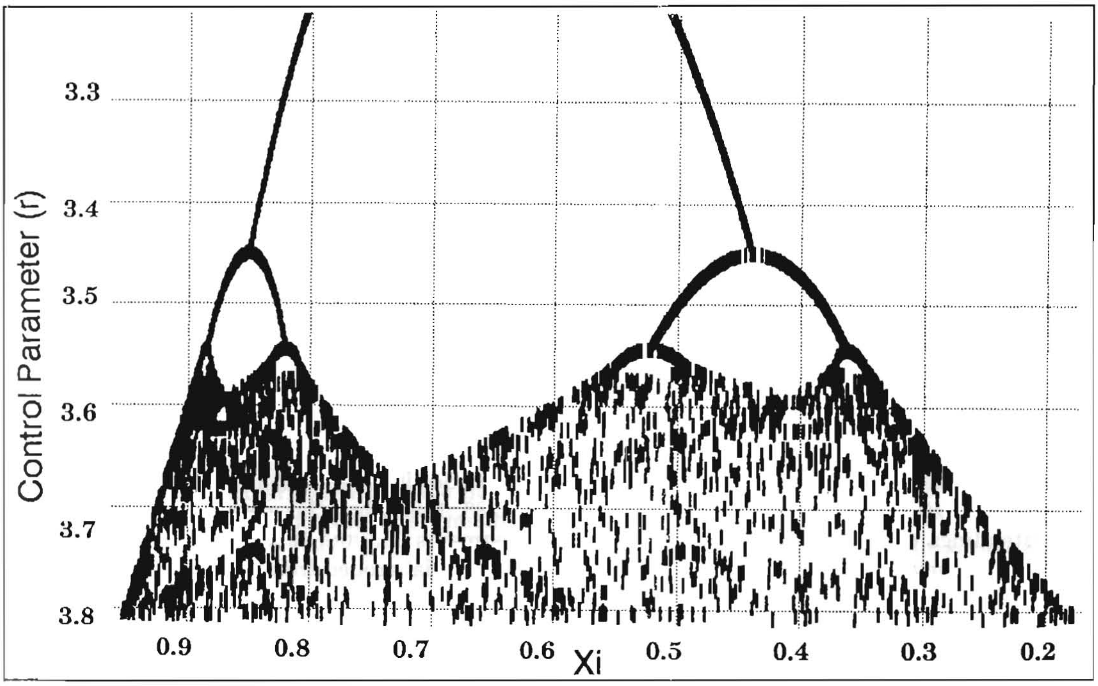


Figure 2. Bifurcation sequence for the logistic equation.

considered to be height, since height controls the velocity of the falling sand grain. Then, one can envisage $X_i$ to be the location where the sand grain comes to rest, which incorporates angle and divergence from the initial condition.

Although clearly the dynamics of erosion have not been fully explained by the preceding interpretation, it is interesting to contemplate the possibility of describing the resulting geometry in terms of deterministic chaos. If it can be described, why then should sand, randomly falling down the face of a dune scarp, follow the universal principal discovered by Feigenbaum? Is the geometry of the dune scarp simply related to the angle of repose or is there a dynamic process occurring worthy of study by coastal scientists and engineers? Is Figure 1 a chance occurrence? Do other coastal processes such as inlet spillover channels exhibit bifurcations? Can the importance of the critical control parameter be useful in predicting channel migration? The answers to questions such as these await exploration based on a new intuition. If certain coastal processes are shown to be deterministic as well as chaotic, findings may lead to an enhanced understanding of the highly complex, coastal environment.

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