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Incipiency of Sediment Motion Under Combined Waves and Currents

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ABSTRACT

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A new formula for nearbottom shear stress under waves and currents was obtained as a result of experiments on the threshold of sediment motion in a laboratory channel. This work supports the hypothesis of the effective shear stress composition in combined flows. A modified longshore sediment transport formula was obtained by adapting the Engelund-Hansen formula in conjunction with the new shear stress formulation. The modified longshore sediment transport formula has a better correlation with the CERC formula than the original Engelund-Hansen formula.

ADDITIONAL INDEX WORDS: *Shear stress, longshore transport, sediment, effective stress, coastal zone.*

INTRODUCTION

There are several total longshore sediment transport formulae. Computational comparisons of these formulae have been discussed in several papers. In GRAFF and OVEREEM (1979) the results obtained using the CERC formula are compared to the results of various sediment transport formulae. Mean shear stress under waves and currents, τ_{wc}, is introduced in every longshore transport formula. In most cases, the Bijker formula for τ_{wc} was used. This equation is

$$\tau_{wc} = \tau_c[1 + \frac{1}{2}\left(\xi\frac{U_o}{V}\right)^2], \tag{1}$$

where τ_c represents the shear stress of the current, ξ is Bijker's parameter:

$$\xi = C_h\left(\frac{f_w}{2g}\right)^{1/2}, \tag{2}$$

f_w is the bottom friction coefficient according to Jonsson, U_o is the amplitude of the orbital velocity at the bed, V represents the mean current velocity, C_h is the Chezy coefficient and g is the gravitational acceleration. It can be shown that equation (1) is a simple sum of shear stress

$$\tau_{wc} = \tau_c + \bar{\tau}_w, \tag{3}$$

where $\bar{\tau}_w$ = mean wave shear stress,

$$\bar{\tau}_w = \frac{1}{4}\rho f_w U_o^2 \quad \text{and } \tau_c = \frac{\rho g V^2}{C_h^2}, \tag{4}$$

ρ is the mass density of water.

Note that the formulae for longshore sediment transport have adapted the formulae for channel sediment transport with τ_c. Formula (1) shows the increase of sediment transport due to waves as a result of an increase in the bottom shear stress.

In the present communication, the modified Engelund-Hansen formula for longshore sediment transport is introduced. It includes both current bottom shear stress and effective wave shear stress. Comparative computations have been carried out with the CERC formula similar to those of GRAFF and OVEREEM (1979). The modified formula shows better correlation with the CERC formula.

BOTTOM SHEAR STRESS UNDER WAVES AND CURRENT

The main conception of the problem's solution is as follows. LENHOFF (1982), NIELSEN (1979), WANG and SHEN (1985), and others showed that the introduction of the effective shear stress in waves gives the direct connection between waves and unidirectional flows. Shields criterion is commonly used to predict the threshold of sediment motion in unidirectional flow. This criterion is used for combined flows with effective shear stress. According to WILLIS (1978)

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$$\tau_{wc} = \tau_c + W\bar{\tau}_w, \qquad (5)$$

with W = 0.6. Parameter W was obtained on the basis of only one experiment, so there is a large probability of error. In eq. (5), the value of τ_{wc} is the absolute one of the resultant shear stress and it does not depend on the angle between the waves and current.

So, for known waves and current parameters at the threshold value, one can calculate the values τ_c and $\bar{\tau}_w$, and then determine τ_{wc} and the coefficient W from Shields diagram. Experimental data on the threshold of sediment movement in combined flows are necessary for this method of approach.

LARSEN *et al.* (1981) conducted field investigations at instrumented tripods on the continental shelves of Australia and the United States. Two of the sites were located at 90 m depth, the third site was located at a depth of 75 m. Investigations included measurements of continuous tidal-current velocity, tidal elevation, wave state, nearbottom sediment concentration and also bottom photography. At two of the sites sediments were represented by sandy silt (D = 0.003–0.012 cm), while at the third site it was fine sand (D = 0.012–0.025 cm), D is a median grain diameter. Determination of threshold conditions is based on the water turbidity time-series plot analysis. When the water turbidity suddenly increased above previous level, the conditions present were used as the threshold state. The bottom state photographs were used for supplementary information.

Our experimental investigations have been carried out in a hydraulic channel having a discharge capacity of 1 to 200 l/sec. To generate regular waves the channel was equipped with a portable wavemaker of plunger type. The generated wave period was 0.67 sec. Transducers were mounted for measurement of wave and current parameters. Micro-hydrometric current-meters with a 4 mm blade diameter and capacitance wave-gauges were used. The measurement/data processing was realized using a computer. As a drift material, particles of various density and size were used. Sediment data are presented in Table 1.

By visual inspection, the following stages of sediment particle behavior have been distinguished: (a) occasional separation of sediment particles, (b) inception of sediment bulk movement (slight transport), (c) inception of mass sediment bulk movement (moderate transport), and (d) mass movement of sediments (intensive transport).

Table 1. *Characteristics of the sediment materials tested.*

Type of Material	Material Density (t/m^3)	D_{50} (m)	Non-uniformity Ratio (D_{60}/D_{10})
Silicon carbide, green	3.20	0.00020	1.26
Silicon carbide, black	3.20	0.00051	1.26
Sand	2.65	0.00070	1.66
Electrocorundum, normal	3.90	0.00082	1.80
Electrocorundum, normal	3.90	0.00129	1.55
Sand	2.65	0.00202	1.28
Sand	2.65	0.00227	1.18
Fine pebble	2.65	0.00412	1.15

During the experiments the sediment samples were placed in a special rack positioned at the channel double bottom. Immediately behind the rack a grit-chamber is fitted, designed to trap the stirring particles. Presence of such a grit-chamber allows observation of the sediment transport intensity when studying the above mentioned stages discerned in the particle behavior. The sediment specific discharge is measured by weighing the particles trapped in the grit-chamber.

The tests were carried out at channel water depth of 15 to 30 cm. The wave height varied within the range of 5–7 cm while the wave length varied between 60 and 70 cm. Observations of the critical conditions of the bottom particles were restricted to 15–20 min. Because of the subjective method of approach used to define the incipient sediment motion and because of the high degree of spread of the experimental points, the same procedure has been applied to a series of experiments aimed at the investigation of the incipient sediment motion in unidirectional current. In the latter case, the same hydraulic channel and exactly the same sediment samples have been examined as in the combined flow.

The calibration of the coefficient W was performed on the basis of comparison of the results obtained for combined flows with the results of incipient sediment motion in unidirectional currents. As a basis for comparison, the specific discharge q_s was taken and was determined by weighing the sediment trapped in the grit-chamber at the end of each experiment.

The values of the sediment specific discharge $q_s \cdot 10^3$ (g/cm sec), measured for the above-mentioned stages of sediment threshold, are presented in Table 2 and Figure 1 near the circles. The black

Table 2. *Measured sediment specific discharge for the sediment threshold under combined waves and current action.*

Median Sediment Grain Diameter (D_{50}) (m)	Mean Current Velocity (V) (m/s)	Water Depth (h) (m)	Wave Height (H) (m)	Sediment Specific Discharge ($q_s \cdot 10^3$) (g/(cm sec)
0.00020	0.33	0.178	0.050	1.66
	0.36	0.185	0.048	5.56
	0.38	0.190	0.047	22.50
	0.41	0.190	0.045	55.27
	0.41	0.217	0.065	48.94
0.00051	0.33	0.174	0.046	0.30
	0.34	0.201	0.067	0.34
	0.35	0.180	0.046	0.63
	0.37	0.188	0.043	1.92
	0.40	0.195	0.045	13.18
	0.46	0.206	0.043	48.54
	0.43	0.200	0.047	36.92
0.00082	0.38	0.182	0.053	0.05
	0.41	0.188	0.054	0.14
	0.44	0.194	0.056	1.56
	0.47	0.201	0.047	3.00
	0.49	0.208	0.049	24.90
	0.51	0.214	0.054	42.42
0.00129	0.46	0.182	0.053	0.43
	0.50	0.188	0.057	0.94
	0.53	0.194	0.050	12.33
	0.55	0.199	0.060	17.43
	0.58	0.205	0.054	43.15
0.00202	0.38	0.163	0.046	0.18
	0.41	0.169	0.051	5.33
	0.45	0.174	0.059	9.50
	0.47	0.180	0.060	33.63
	0.49	0.185	0.055	45.39
0.00227	0.40	0.170	0.052	0.26
	0.48	0.185	0.055	16.90
	0.51	0.201	0.054	26.03
	0.55	0.201	0.054	73.63
	0.42	0.153	0.043	0.70
	0.46	0.163	0.047	16.58
	0.50	0.173	0.042	30.71
	0.56	0.183	0.047	128.30
0.04120	0.62	0.137	0.062	1.33
	0.81	0.137	0.053	150.00
	0.64	0.125	0.075	5.53
	0.70	0.135	0.073	40.43

circles correspond to the unidirectional flow conditions, the white circles correspond to the waves on a current. For some values of D_* the measurements are repeated, for others it was difficult to distinguish between neighbour stages. The results show that for both unidirectional flow and waves on a current the approximately equal values of small sediment discharges are correlated with the same stages of sediment threshold. So, the values of q_s may be used as a worthy objective criterion for the sediment threshold.

To picture the unidirectional flow data on the Shields diagram in coordinates Ψ, D_* (Figure 1) the nearbottom shear stress $\tau = \tau_c$ was obtained using eq. (4) for each flow regime. To picture the waves on a current data on the same diagram, eqs. (4) and (5) were used to obtain nearbottom shear stress $\tau = \tau_{wc}$. As a matter of record for each value D_*, the factor W was obtained in such a way that the sum of the distances along vertical coordinates between constrained points for the unidirectional flow and waves on a current was minimal in the root-mean-square sense. Then the curves of constant sediment discharge (Figure 1) were drawn, the points for both unidirectional flow and waves on a current were taken into account. The linear interpolation between the measured points was used. At the second step the factors W were varied so that the sum of distances between waves on a current points and the q_s-curves was minimal in the root-mean-square sense. Then the q_s-curves were redrawn. The final results of the iterations are shown in Figure 1.

So, the experimental results on combined flows are superimposed upon the results on unidirectional current while the distribution of the sediment specific discharge is kept the same for both current types for the entire range of the sediment critical conditions (from occasional separation of particles to mass sediment movement (Figure 1)).

The parameters which were chosen for graphical presentation of data in Figure 1 are the Shields entrainment function Ψ and a dimensionless grain diameter D_* where

$$\Psi = \frac{\tau}{(\rho_s - \rho) g D}, \qquad (6)$$

$$D_* = \left[\frac{(\rho_s - \rho) g}{\rho \nu^2}\right]^{1/3} D, \qquad (7)$$

ρ_s = mass density of sediment material, ν = kinematic viscosity of fluid, $\tau = \tau_c$ for unidirectional current and $\tau = \tau_{wc}$ for combined flow. The dimensionless grain diameter, D_* can be derived from the drag coefficient and Reynolds number for a settling particle by eliminating the settling velocity.

As a result of the estimation of the empirical coefficient W in accordance with the specific sediment discharge (q_s), it was established that W is not constant and its value depends on the sediment characteristics. A graphical relationship (Figure 2) is derived and is approximated by the expression:

$$W = 0.650 \tanh(0.039 D_*) + 0.060. \qquad (8)$$

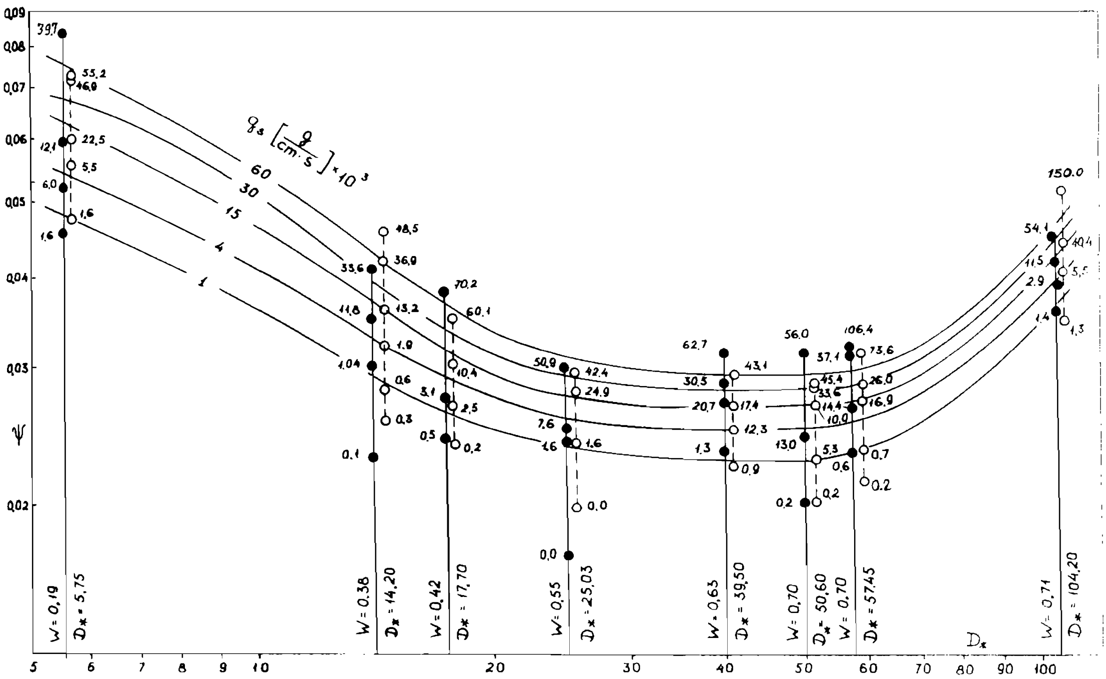

Figure 1. The determination of factor W from the measurements of the specific sediment discharges g_s in g/(cms) in several stages of the threshold for unidirectional flow (●) and for waves and current (○). The curves are the kind of the Shields critical curve in the axis: the Shields entrainment function Ψ versus dimensionless grain diameter D_*.

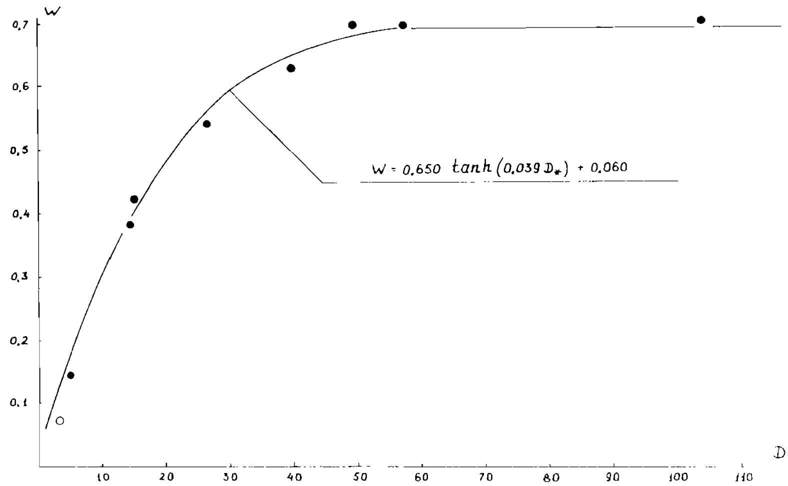

Figure 2. A plot of the factor W versus dimensionless grain diameter D_*: ●—our laboratory experimental data (Figure 1), ○—data of the field measurements, Larsen *et al.* (1981).

To obtain eq. (8), besides our results, some field results of LARSEN *et al.* (1961) have been also used.

Let's discuss this result. For nondimensional diameter D_*, more than about 50, the value of W does not depend on D_* and equals the constant 0.71. It is close to the corresponding value of W = 0.6 (WILLIS, 1978). Then the boundary value $D_* = 50$ corresponds to the boundary of the hydraulically rough region on Shields threshold curve.

The value W = 0.71 may be explained in the following way. Let's determine the effective nearbottom shear stress in waves (NAKAMURA *et al.*, 1970)

$$\tau_{eff} = \rho \frac{f_w}{2} U_{eff}^2, \qquad U_{eff} = \overline{|U_w(t)|}, \tag{9}$$

where U_{eff} = effective wave velocity, equals mean absolute value of bottom wave velocity $U_w(t)$. It is easy to obtain

$$U_{eff} = \frac{2}{\pi} U_o,$$

then

$$\tau_{eff} = \frac{8}{\pi^2} \bar{\tau}_w \approx 0.81 \bar{\tau}_w. \tag{10}$$

So, the value W = 0.71 is close to the value of the factor in effective wave shear stress expression (10). And eq. (5) is the sum of current shear stress and effective wave shear stress.

As the nondimensional diameter decreases, factor W decreases too (Figure 2). The field data of LARSEN *et al.* (1981) verify this behaviour of $W(D_*)$, which shows that the turbulent bursts caused by the steady flow coincide with the wave orbital velocities for fine sand.

It is necessary to note that our experiments were carried out with

$$\frac{U_o}{V} = 0.13, \ldots, 0.3. \tag{11}$$

The sign of additional nearbottom velocity was constant and so the current-meters may be used in the lower part of the flow. Relation (11) corresponds to equal actions of waves and current on a mobile bed. For all calculations the bed roughness, k_b, is equal to the median grain diameter

$$k_b = D \tag{12}$$

Relation (12) was obtained by the standard method from the measured velocity profiles for unidirectional flows. It is valid when the bed is flat and well smoothed (NIELSEN, 1979).

Eqs. (5) and (8) were compared with the known model of bottom boundary layer in combined flow of GRANT and MADSEN (1979). The results of calculation of the Shields function Ψ versus the dimensionless grain diameter D_* for our experimental conditions with minimal specific sediment discharges (about 1 g/(cm sec)) are illustrated in Figure 3. Both curves are related for waves on a current and parameter Ψ was calculated with τ_{wc}. Curve 1 was calculated with formulas (5) and (8) and curve 2 with the model by GRANT and MADSEN (1979). The high correlation confirms validity of eq. (5) also. The disagreement between the two curves for small numbers D_* is in need of subsequent examination.

LONGSHORE SEDIMENT TRANSPORT RATE

In the most common case, the formulae for longshore sediment transport are obtained from corresponding formulae for river and channel sediment transport by replacing the shear stress τ_c with the shear stress for combined flow, τ_{wc}. The adapted Bijker, Engelund-Hansen and Ackers-White formulae were obtained in this way using the Bijker equation for τ_{wc} and were analysed by GRAFF and OVEREEM (1979).

In ACKERS (1983), the comparative computations have been carried out with the number of channel sediment transport formulae and 840 results of laboratory experiments with sand, 180 results of experiments with fine pebbles and 240 data of field measurements. The Ackers-White method has shown the best correlation with field and experimental data. That is, the ratio of calculated and measured sediment loads is in the range 0.5–2.0 for 68% calculations. The similar correlation factor equals 63% for simple Engelund-Hansen method.

For this reason, we shall use the Engelund-Hansen method in the following way. Note that all our conclusions are also valid for the Ackers-White method. The formula of the Engelund-Hansen method is

$$Q_{EH} = V \frac{0.05 C_h \tau^2}{\rho^2 g^{5/2} \Delta^2 D}, \tag{13}$$

where Q = sediment transport in m^3/m; V = mean current velocity; τ = nearbottom shear stress; Δ = relative apparent density of bed material

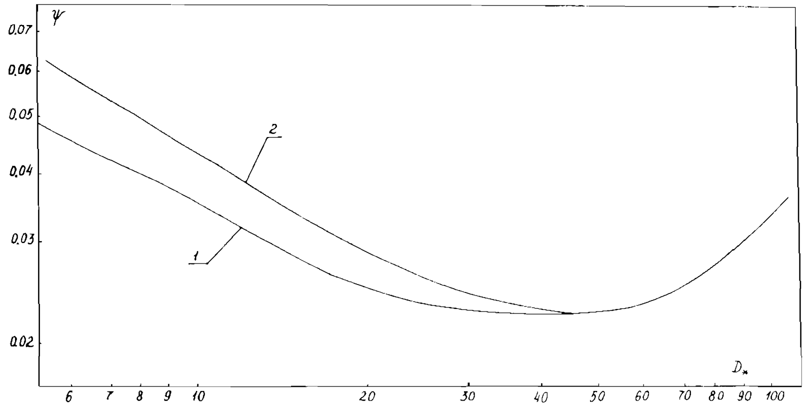

Figure 3. Shields curves for waves on a current (specific sediment discharges about 1 g/(cm sec)) calculated with formulas (5) and (8) (curve 1) and with model by Grant and Madsen (curve 2).

$$\Delta = \frac{\rho_s - \rho}{\rho}.$$

We used eq. (13) with τ_{wc} which was calculated from formula (5). The longshore current velocity was found by BATTJES (1974) for irregular waves. The verified description of local wave parameters in the surf zone due to irregular wave field was available; Battjes introduces fictitious wave heights which can be calculated from shoaling and refraction. The effect of lateral friction in LONGUET-HIGGINS (1970) force balance equation has been included. The computational programme (Fortran-77) has given the smoothed velocity distribution along the coastal slope.

Comparative computations similar to those of GRAFF and OVEREEM (1979) but with modified Engelund-Hansen formula (MEH) have been carried out under various boundary conditions. The mean wave height on deep water varied from 0.5 to 3.0 m; the wave period from 4 to 8 sec; the angle of wave incidence from 10° to 80°; the wave breaking index from 0.4 to 1.0; the sediment particle diameter from 0.1 to 0.3 mm; the slope of the beach profile from 1:100 to 1:20; and finally, bed roughness from 0.02 to 0.10 m. The wide range of boundary conditions gives a complete picture of the new formula results behaviour.

At first the wave shoaling and refraction from deep water to breaker line were calculated by Airy ray theory. The total sediment transport was rated by the CERC formula

$$Q_{CERC} = AH_o^2C_o\sin\varphi_b\cos\varphi_o, \tag{14}$$

where A = dimensionless coefficient, A = 0.042 if

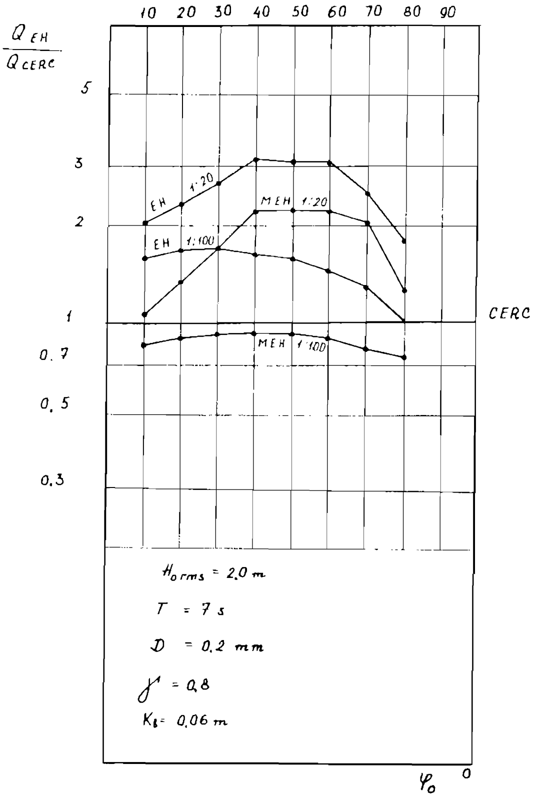

Figure 4. Sediment transport computations with the modified Engelund-Hansen formula (MEH) and original Engelund-Hansen formula (EH) and in comparison to CERC formula. The effect of the deep water wave incidence, φ_o; T is the wave period, γ is the wave breaking index.

$H_o = H_{orms}$ (GRAFF and OVEREEM, 1979), H_o is the deep water wave height; C_o is the deep water wave celerity; φ_b is the angle of wave incidence at the breaker line; φ_o = idem in deep water. Then the sediment transport distribution were rated by modified Engelund-Hansen formula with the longshore velocity distribution from BATTJES (1974) model. Finally, the total sediment transport Q_{MEH} was calculated by integration.

The calculations were carried out with W = 1, that was in accordance with Bijker formula for τ_{wc} and GRAFF and OVEREEM (1979) results. Then, the calculations were carried out with W = 0.71 from our results. We don't introduce the decrease of factor W for the fine material because the conditions in which formula (8) was obtained, flat and smooth bed, differ from the conditions of field beach profile with bed forms.

The example of calculations is shown in Figure 4 with effect of deep water wave incidence angle. The modified Engelund-Hansen formula (MEH) has a better correlation with the CERC formula than the original Engelund-Hansen formula (EH). Similar results are obtained by varying other factors: wave height, wave period, wave breaking index and so on, and by using the modified Ackers-White formula.

CONCLUSIONS

The new formula for the effective nearbottom shear stress under combined waves and currents was obtained as a result of the experiments on incipient sediment motion. The onset of sediment motion used as an indicator of a level of the critical shear stress and the value of factor W in the wave term was obtained with the help of Shields critical curve for unidirectional flow. It was shown that this factor, W, is not constant and its value depends on the sediment characteristics.

The Engelund-Hansen formula on the longshore sediment transport with shear stress τ_{wc} gives better correlation with the known CERC formula than the original Engelund-Hansen formula. It may be used in different problems relating to sediment transport by waves and currents. More field data are needed to confirm the validity of the new equation and this is the main problem for future research.

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LITERATURE CITED

ACKERS, P., 1983. Sediment transport problems in irrigation system design. *In: Developments in Hydraulic Engineering*—1. New York: Applied Science Publishers, pp. 151–195.

BATTJES, J.A., 1974. Computation of set-up, longshore currents, run-up and overtopping due to wind-generated waves. Delft University of Technology, *Communications on Hydraulics, No. 74-2,* Department of Civil Engineering.

BIJKER, E.W., 1967. Some considerations about scales for coastal models with movable bed. Delft Hydraulics Laboratory, *Publication No. 50,* Delft.

GRAAF, J. and OVEREEM, J., 1979. Evaluation of sediment transport formulae in coastal engineering practice. *Coastal Engineering,* 3, 1–32.

GRANT, W.D. and MADSEN, O.S., 1979. Combined wave and current interaction with a rough bottom. *Journal of Geophysical Research,* 84, 1797–1808.

LARSEN, L.H.; STERNBERG, R.W.; SHI, N.C.; MARSDEN, M.A.H., and THOMAS, L., 1981. Field investigations of the threshold of grain motion by ocean waves and currents. *Marine Geology,* 42, 165–182.

LENNOFF, L., 1982. Incipient motion of particles under oscillatory flow. *Proceedings 18th Coastal Engineering Conference* (Cape Town), Vol. 2, pp. 1555–1567.

LONGUET-HIGGINS, M.S., 1970. Longshore currents generated by obliquely incident sea waves. *Journal of Geophysical Research,* 75, 6778–6789.

NAKAMURA, M.; SHIRAISHI, H.; SASAKI, Y., and ITO, M., 1970. Hydraulic study of sand transportation caused by waves progressing in current. *Bulletin of Agricultural Engineering Research,* 7, 209–230 (in Japanese).

NIELSEN, P., 1979. Some basic concepts of wave sediment transport. Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark, *Series Paper 20,* 160p.

WANG, S.-Y. and SHEN, H.W., 1985. Incipient sediment motion and riprap design. *Journal of Hydraulic Engineering,* 111(3), 520–538.

WILLIS, D.H., 1978. Sediment load under waves and currents. *Proceedings 16th Coastal Engineering Conference* (Hamburg), Vol. 2, pp. 1626–1637.

□ RÉSUMÉ □

A partir d'expériences en cuve à houle sur le seuil de mise en mouvement des sédiments, on a obtenu une nouvelle formule de la force de cisaillement à proximité du fond sous l'action de la houle et des courants. Ce travail appuie l'hypothèse de la composition de la force de cisaillement efficace dans des flux combinés. On a obtenu une équation modifiée du transport sédimentaire parallèle à la côte en adaptant la formule de Engelung-Hansen à la nouvelle formulation de la force de cisaillement. La nouvelle formule du transport sédimentaire parallèle à la côte est mieux correlée avec celle du CERC que la formule originelle de Engelund-Hansen.—*Catherine Bousquet-Bressolier, Géomorphologie, E.P.H.E., Montrouge, France.*

□ ZUSAMMENFASSUNG □

Das Ergebnis von Laborexperimenten über Beginn bzw. Schwellenwert von Sedimentbewegungen in einem Kanal ist eine neue Formel für die bodennahe Scherbeanspruchung. Diese Untersuchung unterstützt die Hypothese zur Zusammensetzung des effektiven Scherstresses in kombinierten Strömungen. Eine modifizierte Formel des küstenparallelen Sedimenttransportes erhielt man bei der Anwendung bzw. Anpassung der ENGELUND/HANSEN-Formel in Verbindung mit der neuen Scherstressformel. Die modifizierte Gleichung zum Sedimenttransport weist eine höhere Korrelation mit der CERC-Formel als mit der ursprünglichen ENGELUND/HANSEN-Formel auf.—*Ulrich Radtke, Geographisches Institut, Universität Düsseldorf, F.R.G.*