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Ground Water in Barrier Islands—Theoretical Analysis and Evaluation of the Unequal-Sea Level Problem

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ABSTRACT

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The Ghyben-Herzberg lens in a strip island is symmetric with respect to the centerline of the island, if recharge and hydraulic conductivity are uniform across the island and effective sea level (*i.e.*, the level of ground-water outflow) is the same on both sides of the island. The unequal- or two-sea level problem results when effective sea level is higher on one side, as would be the case in some barrier islands because of wave runup and the effect of a sloping beach on water-table tides. An approximate analytical solution is available for the cross-island variation of water-table elevation and depth to interface in an unequal-sea level island, but derivation of the water-table position in this solution makes the usual Ghyben-Herzberg assumption that salt-water head is zero. This assumption is not valid here because salt-water head must vary across the island in association with a salt-water flow that would be driven by the unequal sea levels.

A finite-difference solution for the configuration of the two-sea level lens is developed here. The model takes into account the across-island variation in salt-water head. The model handles the resulting nonlinear differential equation with an iterative scheme in which the Thomas Algorithm is used to solve the finite-difference equations during each iteration. Results of a parametric study of the asymmetry of the lens vs magnitude of sea-level difference indicate: (1) use of the approximate analytical solution that ignores the nonzero saltwater head can produce significant errors, especially with respect to the volume of the lens, and (2) the effect of the difference in effective sea levels is apt to be overshadowed by the effects of nonuniform recharge and/or hydraulic conductivity. The latter effects become relatively more important in islands that are larger, more highly recharged, and/or have a lower overall hydraulic conductivity.

ADDITIONAL INDEX WORDS: *Ghyben-Herzberg lens, island hydrology, hydrogeology, finite-difference models, beach tides.*

INTRODUCTION

In small islands, fresh ground water typically occurs in Ghyben-Herzberg lenses, which are bodies of fresh ground water floating on more dense, salty ground water. The fresh ground water is from local, on-the-island recharge. The underlying salty ground water is of marine derivation. The contact between the fresh and salty ground water is referred to as an "interface", although, in fact, there is a dispersion-related transition zone between the two fluids.

The Ghyben-Herzberg Principle relates the elevation of the water table above sea level to the depth of the interface below sea level. According to the original formulation by Badon Ghyben and Herzberg (*e.g.*, TODD, 1980), the depth to the interface is simply 40 times the water-table elevation (where the "40" assumes the usual densities of fresh water and seawater). HUBBERT (1940) derived a more general relationship, which reduces to the simple Ghyben-Herzberg Principle for the case of horizontal flow and stationary sea level (zero salt-water head). Field studies (TODD and MEYER, 1971; VACHER, 1978; SCHMORAK and MERCADO, 1979) have indicated that the variation of salinity through the transition zone is symmetric, so the iso-surface of 50% relative salinity can be taken as the position of the interface for the purpose of applying Ghyben-Herzberg-type relationships.

For long, narrow, so-called "infinite-strip" islands—in which island width, recharge and hydraulic conductivity do not vary along the long dimension of the island—the geometry of the Ghyben-Herzberg lens is described by a vertical, shore-to-shore cross section normal to the shoreline (Figure 1). If both recharge and

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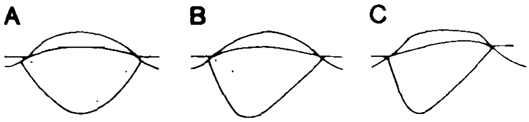

Figure 1. Variations in the shape of Ghyben-Herzberg lenses: A, symmetric lens as when recharge and hydraulic conductivity are constant and effective sea level is the same on both sides of the island; B, asymmetry due to variation across island of recharge or hydraulic conductivity; C, asymmetry due to unequal effective sea levels.

hydraulic conductivity are constant across the width of the island, the lens is symmetric about an axial divide along the centerline of the island (Figure IA). If either recharge or hydraulic conductivity varies across the island (Figure 1B), the lens is asymmetric (VACHER, in press). If there is an across-the-island variation in hydraulic conductivity, the lens is skewed toward the more impermeable side of the island. If there is an across-the-island variation in recharge, the lens is skewed toward the more highly recharged side of the island. In both cases, the flow divide (highest water table) occurs at the location of the greatest thickness of the lens. This is a consequence of the Ghyben-Herzberg Principle.

URISH (1980) has pointed out that lens asymmetry will also be produced, if effective sea level is higher on one side of the island than on the other. In this case (Figure IC), there are disparate levels of groundwater discharge. As a result, the lens is distorted; the flow divide is shifted toward the high-sea level side of the island, and the location of the deepest interface is shifted in the opposite direction, toward the low-sea level side of the island. This apparent contradiction of the Ghyben-Herzberg Principle is resolved by remembering that salt-water head is not constant in this case. Marine-derived ground water must flow, beneath the lens, from the high-sea level side to the low-sea level side of the island, so the appropriate relationship between water-table elevation and depth to interface is that of HUBBERT (1940).

URISH (1980) presents an approximate analytical solution defining the water table and interface in a strip island with unequal sea levels (the "two-sea level problem"). The solution for the water table follows from the usual Ghyben-Herzberg Principle. This solution is then combined with the more generalized relation from HUBBERT (1940) to give the solution for the interface.

The purpose of this paper is two-fold: (1) to re-examine the two-sea level problem using the more appropriate HUBBERT (1940) relationship for both the water table and interface simultaneously; and (2) to compare the magnitude of the lens asymmetry resulting from unequal sea levels to that produced by across-island variations in hydraulic conductivity and recharge. Incorporation of the HUBBERT (1940) relationship into the analysis makes the governing differential equation nonlinear, so a finite-difference model is developed to treat the two-sea level lens. The model can also be applied to one-sea level problems that result from lateral gradations in hydraulic conductivity or recharge.

HABITAT OF THE TWO-SEA LEVEL LENS

URISH (1980) indicates that barrier islands are likely examples of islands with asymmetric lenses of the type shown in Figure IC. According to URISH (1980), the elevation of outflow from the lens is effectively higher on the oceanside of the barrier because of wave runup and the effect of a sloping beach on the groundwater tides. His field example is East Beach along Block Island Sound, Rhode Island. At this island, which is 250 m wide, the flow divide and location of greatest thickness occur at 55 m and 170 m, respectively, from the ocean beach, and he found a higher effective sea level on the ocean side.

The effect of wave runup is illustrated in the studies of tidal fluctuations in beaches by HARRISON *et al.* (1971) and DOMINICK *et al.* (1971). Both studies indicate that tide levels measured in the beach are higher than in still-water gauges. Both groups of investigators attribute the difference to the effect of waves superimposed on the tidal fluctuation. In particular, HARRISON *et al.* (1971) include a diagram (their Figure 2) that shows the high-tide and low-tide water tables grading to the swash limit, which is above the still-water position of high and low tide.

An additional mechanism for elevating the water table in a sloping beach was presented by URISH (1980). As explained by him, the water table at the edge of a sloping beach reaches a maximum with the maximum of ocean tide, but, because ground water must drain out as the "shoreline" recedes, the beach water table lags behind sea level during falling tide. Therefore, mean beach level is higher than mean ocean level. The difference varies inversely with beach slope. So, if the tides are significantly diminished in the lagoon, and there is a low-angle beach on the ocean side, the effective sea level on the ocean side would be higher than on the lagoon side. This phenomenon of the lagging beach water table, incidentally, may account for the commonly observed seepage from the low-intertidal beach during low tides.

ANALYSIS

Background

As shown in Figure 2, the two-sea level problem is one of a family of closely related problems of steady-state flow in phreatic (*i.e.* , water-table) aquifers. Case I represents flow between parallel, fully-penetrating rivers; Case II covers the usual, one-sea level Ghyben- Herzberg lens; Case III treats the two-sea level Ghyben-Herzberg lens. In each case, the problem is to find head as a function of distance from one of the boundaries. The general approach in such phreatic problems is to make the problems one-dimensional by use of the Dupuit assumptions—that head is a function of lateral position only (*i.e.* , vertical equipotentials, no vertical components of hydraulic gradient, horizontal flow).

The solutions for I and II are given in the following references: TODD (1980) and WANG and ANDERSON (1982) for Case IA; FETTER (1980) for Case IB; TODD (1980) and FETTER (1980) for Case II. The solutions provide perspective on the peculiarities of Case III. The general method of solution employs the continuity equation (conservation of mass) and Darcy's Law to a vertical slice of the aquifer (Figure 2), and is outlined in the following, adapting procedures for Dupuit-type problems in WANG and ANDERSON (1982).

The continuity equation (that inflows − outflows = 0 in the steady- state) for the slice is:

$$Q\Big|_{x+\Delta x} - Q\Big|_{\Delta x} = R\,\Delta x \qquad (1)$$

where the Q's refer to total fluxes and R is

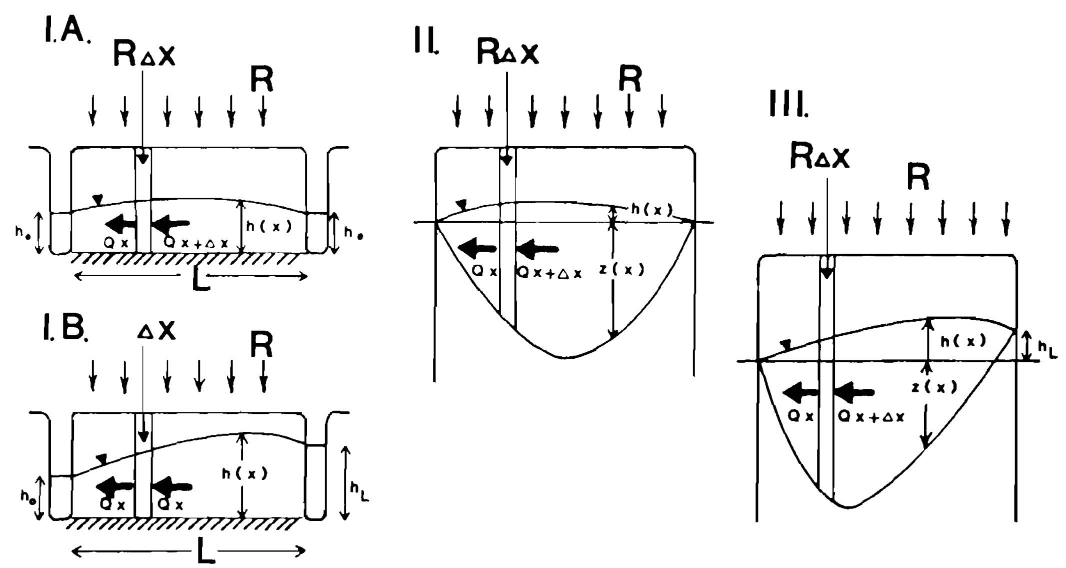

Figure 2. One-dimensional water-table problems: I, flow between parallel ditches with equal (A) and unequal (B) boundary heads; II, Ghyben-Herzberg lens with equal effective sea levels; III, Ghyben-Herzberg lens with unequal effective sea levels.

recharge (assumed uniform for these problems). Darcy's Law at the sides of the slice is:

$$Q\Big|_{x+\Delta x} = -[KA\,(dh/ds)]\Big|_{x+\Delta x} \quad \text{and}$$

$$Q\Big|_{x} = -[KA\,(dh/ds)]\Big|_{x} \qquad (2)$$

where K is hydraulic conductivity, A is the cross section of flow, and dh/ds is the hydraulic gradient (in the direction of flow).

For Case I, the Dupuit assumptions allow A and dh/ds in equation (2) to be replaced with h and dh/dx. For Case II, the Dupuit assumptions mean that A and dh/ds can be replaced with (h+z) and dh/dx. In addition, for Case II, z can be replaced by αh because of the Ghyben- Herzberg Principle,

$$z = \alpha h, \ \alpha = \rho_f / (\rho_s - \rho_f) \qquad (3)$$

where ρ_f and ρ_s are the densities of the fresh and underlying salty ground water. Thus with these substitutions, and replacing h (dh/dx) by $[d(h^2)/dx]/2$, equation (1) becomes:

$$\text{Case I } [Kd(h^2)/dx]\Big|_{x+\Delta x} - [Kd(h^2)/dx]\Big|_{x} = -2R\Delta x \qquad (4a)$$

$$\text{or } [d(h^2)/dx]\Big|_{x+\Delta x} - [d(h^2)/dx]\Big|_{x} = -R\Delta x/K \qquad (4b)$$

$$\text{Case II } [Kd(h^2)/dx]\Big|_{x+\Delta x} - [Kd(h^2)/dx]\Big|_{x} = -2R\Delta x/(\alpha+1) \qquad (5a)$$

$$\text{or } [d(h^2)/dx]\Big|_{x+\Delta x} - [d(h2)/dx]\Big|_{x} = -2R\Delta x/K(\alpha+1) \qquad (5b)$$

where (4a) and (5a) allow for lateral variation in K, and (4b) and (5b) apply to homogeneous aquifers. Then the corresponding flow equations, which result from letting Δx become infinitesimally small, are:

$$\text{Case I } d/dx[Kdh^2/dx] = -2R \quad [\text{heterogeneous}] \qquad (6a)$$

$$d^2h^2/dx^2 = -2R/K \quad [\text{homogeneous}] \qquad (6b)$$

$$\text{Case II } d/dx[Kdh^2/dx] = -2R/(\alpha+1) \quad [\text{heterogeneous}] \qquad (7a)$$

$$d^2h^2/dx^2 = -2R/K(\alpha+1) \quad [\text{homogeneous}] \qquad (7b)$$

Analytical solutions of homogeneous cases are found by integration of equation (6b) or (7b) with appropriate boundary conditions. For Case IA and h = 0 at both x = 0 and x = L,

$$h^2 = R\,(Lx - x^2)/K \qquad (8)$$

For Case IB, where $h = h_0$ at $x = 0$ and $h = h_L$ at $x = L$,

$$h^2 = h_o^2 + (h_L^2 - h_o^2)(x/L) + (R/K)\,(Lx - x^2) \qquad (9)$$

For Case II, where h = 0 at both x = 0 and x = L,

$$h^2 = R\,(Lx - x^2)/K\,(\alpha + 1) \qquad (10).$$

From equation (10), the depth (z) to the interface is known immediately from the Ghyben-Herzberg Principle. Alternatively, an equation for z can be derived from a boundary condition assuming an outflow face (derived independently from potential theory), but the differences are generally not significant (VACHER, in press).

The Two-Sea Level Problem

URISH (1980) gives an approximate solution for the two-sea level problem as:

$$h^2 = h_L^2\,(x/L) + R\,(Lx - x^2)/K(\alpha + 1) \qquad (11).$$

where h = 0 and x = 0 is at the lagoon shoreline, and $h = h_L$ and x = L is at the ocean shoreline.

This solution from URISH (1980) should be compared with the Case IB solution (equation 9) for $h_0 = 0$. The two differ by only the insertion of the Ghyben-Herzberg factor $(\alpha + 1)$; that is, the equation given by Urish is the solution for Case II (equation 7b) for boundary conditions of h = 0 at x = 0 and $h = h_L$ at x = L. However, equation (7b) is not an appropriate flow equation for the two-sea level problem, because that equation is based on the simplified Ghyben-Herzberg Principle that assumes the salt water is stationary.

In order to accommodate the presence of salt-water flow beneath the lens, the flow equation for Case II must incorporate the expression from HUBBERT (1940) for the relation of depth to interface and elevation of the water table:

$$z = (\rho_f h - \rho_s h_s)/(\rho_s - \rho_f) \qquad (12),$$

where h_s is the salt-water head, which varies linearly across the island in the two-sea level problem. From equation (12) the cross-sectional area of flow (z + h) in the Dupuit version of Darcy's Law must be replaced with $(\alpha + 1)(h - h_s)$, which makes the resulting differential equation nonlinear. With the substitutions in equation (2), the flow equation for the two-sea level problem becomes:

$$\text{Case III } d/dx[K(h-h_s)(dh/dx)] = -R/K(\alpha+1)[\text{heterogeneous}] \quad (13a)$$

$$d/dx[(h-h_s)(dh/dx)] = -R/K(\alpha+1) \ [\text{homogeneous}] \quad (13a)$$

If $h_s = 0$, then the equations of Case III reduce to those of Case II.

NUMERICAL SOLUTION

Formulation

A one-dimensional finite-difference model was used to solve equation (13) for two-sea level, homogeneous islands, Equation (13a) was used, because the model was used also to solve one-sea level heterogeneous cases ($h_s = 0$, thus equation 7a) for the purpose of comparing the effects of unequal sea levels to those of island heterogeneities.

In finite-difference form, with h(i), h(i-1), and h(i+1) representing h(x), $h(x - \Delta x)$, and $h(x + \Delta x)$, equation (13a) is

$$K_{01} [(h-h_s)_{01}] [h(i) - h(i-1)]/\Delta x - K_{02} [(h-h_s)_{02}] [h(i+1) - h(i)]/\Delta x = R(\Delta x)^2/(\alpha+1) \quad (14),$$

where K_{01} and K_{02} denote the average K between adjacent nodes (x and $x - \Delta x$ for K_{01}, and x and $x + \Delta x$ for K_{02}), and $41(h-h_s)_{01}$ and $41(h-h_s)_{02}$ refer to the flow thickness at $x - \Delta x/2$ and $x + \Delta x/2$, respectively, with $\alpha = 40$.

With $L = n\Delta x$, there are n+1 nodes across the island (including the shoreline boundaries). There are n-1 equations of the form of equation (14).

At the shoreline nodes, $h - h_s = 0$. The saltwater head, h_s, varies linearly from $h_s = 0$ at $x = 0$ to $h_s = h_{sL}$ at $x = L$.

Procedure

The n-1 equations were solved with the Thomas Algorithm using the program listed in WANG and ANDERSON (1982). The coefficients of the tridiagonal matrix were found by rearranging equation (14) as:

$$-(K_{01}b_{01})h(i-1) + (K_{01}b_{01} + K_{02}b_{02})h(i) - (K_{02}b_{02})h(i+1) = R(\Delta x)^2 \quad (15),$$

where the longer expressions for the average flow thicknesses are replaced with b_{01} and b_{02} Following KINZELBACH (1986), b_{01} and b_{02} were taken as the arithmetic mean of the thickness at adjacent nodes [*e.g.*, 41 (h (i) + h (i-i) - h_s (i) -h_s (i-1)) for b_{01})], and K_{01} and K_{02} were taken as the harmonic mean of K at adjacent nodes [*e.g.*, 2K(i) K(i-1)/(K(i) + K(i-1)) for K_{01}]

The coefficients in equation (15) depend on the h's, so an iterative procedure was used in which the n-1 equations were solved by the Thomas Algorithm within each iteration (cf., KINZELBACH, 1986, p. 63). That is, the h's of the $(m+1)^{th}$ iteration were found using the b's calculated from the h's of the $(m)^{th}$ iteration. However, it was found that the computed results would overshoot the solution, so the change in h between successive iterations was decelerated by

$$h_m = \omega h_T + (1 - \omega) h_{m-1}$$

where h^m and h^{m-1} represent the results for the $(m)^{th}$ and $(m-1)^{th}$ iteration, h_T is the result from the Thomas Algorithm solution, and ω is an under-relaxation factor ($\omega < 1$).

The problem was solved for several cases of h_{sL}, R, K, and L. The model was checked against analytical solutions (VACHER, in press) for islands in which $h_{sL} = 0$ and either R or K varied in adjacent strip segments of L. For the two-sea level cases, ω runs with a different number (n) of x increments produced comparable results if n exceeded about 50.

For cases diagrammed here, n was set at 200, so that the location of the flow divide and the position of the deepest interface could be more accurately interpolated. For the large-n, two-sea level cases, needed to be extremely small (0.1 to 0.003, with smaller ω corresponding to larger differences in sea level). The runs, which were performed (uncompiled) on an IBM PC, were made without a convergence limit and were allowed to continue for 5 hrs (large ω) to 3 days (small ω). When the runs were terminated, there was convergence of successive iterations to within 10^{-6} for V/Vc (the ratio of volumes between the modeled island and the one-sea level island with the same R, K, and L). The final result in V/Vc was approached to within 10^{-4} at far fewer iterations (about 100 iterations or 1 hr for small differences in sea level and 2300 iterations or a day for large differences in sea level), at which time the maximum difference between head values of successive iterations (the convergence requirement used by WANG and ANDERSON, 1982) was less than 10^{-5} The result at about half these iterations was within 1% of the final result obtained with the "unlimited" runs, and should be sufficient for most applications.

Runs for which $h_{sL} = 0$ and K graded linearly from one side of the island to the other were not nearly as computationally demanding The under-relaxation factor, ω, was larger (0.5), and the number of iterations were much fewer (about 20).

Definition and Parameters. The results are presented in terms of dimensionless variables defined by comparison of the lens in the two-sea level island with a standard lens (VACHER, in press), which is the one-sea level lens that would occur in an island with the same R, K, and L (see Figure 3). The configuration of the lens is defined by the variation of h/h_{cm} and z/z_{cm} against x/L, where the h and z refer to the water-table elevation and depth to interface, respectively, in the modeled two-sea level island, and h_{cm} and z_{cm} are the centerline h and z in the comparison one-sea level island. The parameter, HLND, is a dimensionless measure of the difference in sea level between the two sides of the island and is defined as h_L/h_{cm}.

The importance of HLND is that the configuration of h/h_{cm} and z/z_{cm} is the same in all two-sea level lenses with the same HLND. That is (see Figure 3), if HLND = 1.0, the maximum depth to interface and the maximum height of the water table occur, respectively, at 44% and 69% of the cross-island width from the lagoon, regardless of R, K, or L. The effect of an increase in L is enlargement of both the one- and two-sea level lenses; the ratios h/h_{cm} and z/z_{cm} do not change. The effect of an increase in R/K is an equal vertical enlargement (*i.e.*, increase in $(h+z)/L$) of both lenses; again, the inter-lens ratios h/h_{cm} and z/z_{cm} are not affected.

Parametric Study. Selected tilted lenses are shown in Figure 4. The limiting case is the symmetric, one-sea level island where HLND = 0. With increasing "sea-level tilt" (HLND), the flow divide shifts toward the high-sea level side of the island. The site of the deepest interface advances toward the low-sea level side of the island.

The progressive divergence of the location of the highest water table (M_h/L) from the location of the deepest interface (M_z/L) is shown for a range of HLND in Figure 5. As shown, this increase in asymmetry is accompanied by a slight decrease in the volume of the lens (V/V_c) as the amount of sea-level tilt increases.

Graphs of M_h/L, M_z/L, and V/V_c as calculated from the approximate solution from URISH (1980) (equation 12) are included in Figure 5. Obviously, there is poor agreement between the results of the finite-difference model and the approximate analytical solution. The reason is that the nonzero salt-water head of the two-sea level island is taken into account in the finite-difference solution and was ignored in the derivation of the approximate analytical solution for h.

The asymmetry and configuration of a lens in a two-sea level island can be more successfully evaluated from Figures 4 and 5 than by use of the approximate analytical solution. These figures make use of the parameter HLND which compares the difference in sea level to the maximum water-table elevation in the equivalent one-sea level island; therefore, HLND is a function of R, K and L. This standardization parameter can be easily evaluated from Figure 6 from the ratio of recharge to conductivity (R/K) and the ratio of the sea-level difference to island width (h_L/L). The band of R/K values indicated on the figure should bracket most natural lenses in islands composed of Holocene clastics or Late Pleistocene calcarenites.

EVALUATION OF THE SIGNIFICANCE OF THE TWO-SEA LEVEL PROBLEM

The finite-difference model allows comparison of two-sea level lenses with asymmetric lenses that result from across-island variations in hydraulic conductivity or recharge.

One such comparison of the three types of asymmetric lenses is shown in Figure 7. The two-sea level geometry is for an island where the difference in sea level is twice the center-

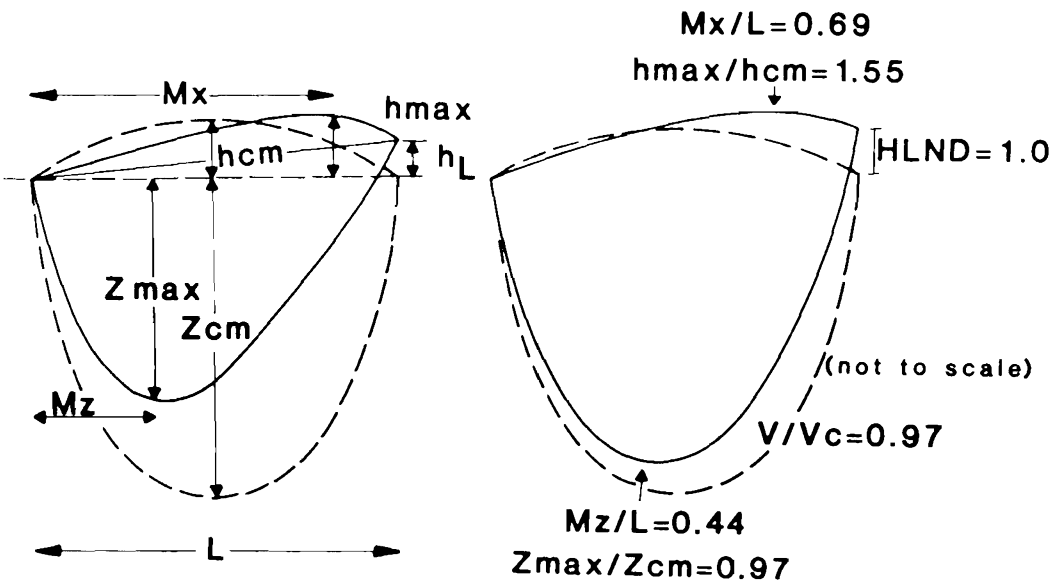

Figure 3. Definition of terms for standardization of the two-sea level lens. The dashed lens is the symmetric lens that would occur in the same island (same R, K and L) if there was no difference in sea level. Standardization of the two-sea level configuration by dividing h and z of the two-sea level lens by, respectively, h_{cm} and z_{cm} of the comparison one-sea level lens means that all two-sea level lenses with the same HLND ($= h_L/h_{cm}$) have the same configuration and z/z_{cm} vs x/L).

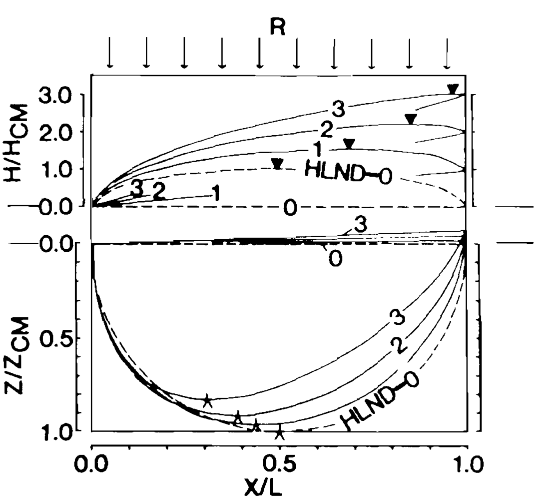

Figure 4. Water table and interface in two-sea level lenses with parameter HLND. The trace of the water table is drawn with a vertical exaggeration of 8 relative to the trace of the interface. HLND = 0 is the one-sea level case (dashed).

line water-table elevation in the comparison island (*i.e.*, HLND = 2). The nonuniform-K lens is for an island in which hydraulic conductivity varies linearly from K_1 on one side of the island to $K_2 = 5K_1$ on the other. The nonuniform-R lens is for an island in which an L/2-wide strip segment adjacent to one shoreline receives recharge R_1, and the other half-island receives recharge $R_2 = 0$. Both one-sea level lenses were calculated with the finite-difference model (with HLND = 0); the nonuniform-R case was also calculated with an analytical solution (VACHER, in press).

The three lenses in Figure 7A are all standardized relative to the symmetric lens that would occur in the same island if K and R were uniform at K_1 and R_1, and there were no difference in sea level. It is clear that the changes in hydraulic conductivity and recharge affect the volume of the lens far more than the difference in sea level. However, the three lenses are practically identical in geometry. This is shown by Figure 7B, in which the three lenses are redrawn with the maximum depth to the interface in each lens set at 1.0. In each of the lenses, the depth to the deepest interface occurs at about x/L = 0.38.

It is easy to envisage the kind of variation in

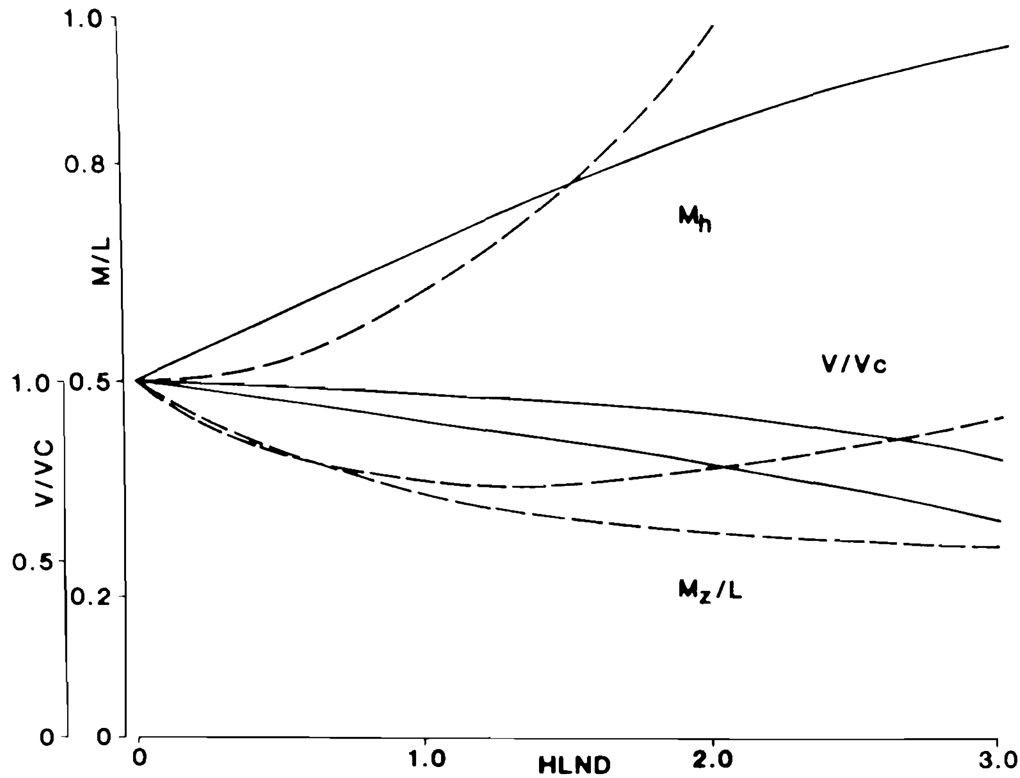

Figure 5. Graphs showing variation of size and asymmetry of two-sea level lenses as function of HLND. Solid lines are from finite-difference model (this paper); dashed lines are from approximate analytical solution (URISH, 1980). M_h/L and M_z/L refer to position of maximum water table and deepest interface, respectively. V/V_c is volume of two-sea level lens relative to volume of the comparison one-sea level lens.

K and R assumed in Figure 7, especially for barrier islands. A 5-fold decrease in K from one side of the island to the other would be consistent with either a lagoonward decrease in grain size or a lagoonward increase in the fraction of the column composed of lower-K beds. Regarding the effect of grain size, standard correlation curves (MASCH and DENNY, 1966) suggest a 4-fold difference in K between well-sorted medium-grained sand and poorly sorted fine-grained sand (*i.e.*, 32 m/day for sand with median grain size of 1.5ϕ and inclusive sorting coefficient of 0.25ϕ, and 8 m/day for sand with median grain size of 2.5ϕ and inclusive sorting coefficient of 0.75ϕ). For the alternative model, where a gradation in K reflects a lagoonward increase in relative abundance of low-K beds, a 5-fold difference in effective hydraulic conductivity of a column consisting of interlayered 50-m/day and 5-m/day sediment would result, for example, if the aggregate thickness of the lower-K material increased from 23% to 93%. In either of these cases, the skew of the interface would be the same as that caused by a "sea-level tilt" of HLND = 2. The direction would be the same, also, if the high effective sea level were on the ocean side of the island, as would be consistent with the higher wave energy and commonly larger tidal range that occurs on the ocean side of barrier islands.

A nonuniform distribution of recharge could arise in a variety of ways. For example, recharge may be concentrated on the ocean side of the island because of the dune ridge and its lack of vegetation and soil cover. In the extreme (R_2 = 0), this would result in a skew of the interface such as that shown in Figure 7. The direction of skew would be opposite that from a

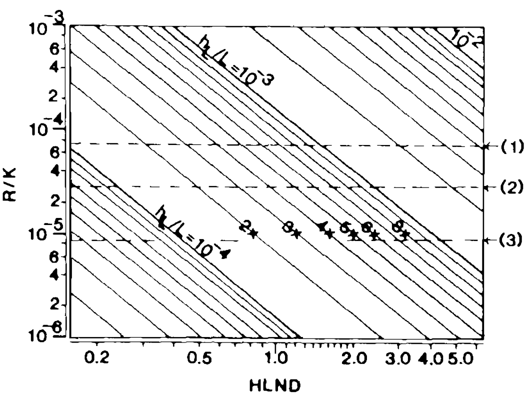

Figure 6. Conversion graph relating R/K and h_L/L of a two-sea level lens to the inter-island parameter HLND which is used in the other figures. Lines at (1), (2), and (3) indicate estimated R/K at, respectively, East Beach, R.I. (URISH, 1980), South Fork, Long Island, N. Y. (FETTER, 1972), and Late Pleistocene calcarenites, Bermuda (VACHER, 1978).

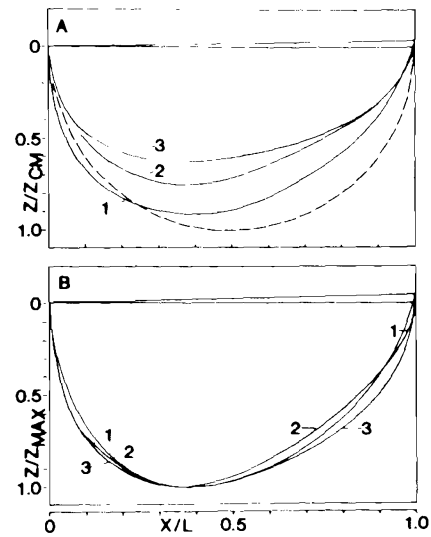

Figure 7 Calculated cross sections comparing the interface asymmetry due to three different causes: (1) a difference in sea level (where HLND = 2); (2) across-island gradation in hydraulic conductivity (where K on one side is 5 × the K on the other side); and (3) strips of different recharge (where there are 2 strips, each half the width of the island, and one of them receives R = 0). In A, the interface depth is standardized relative to z_{cm} of the comparison one-sea level island. In B, the interface depths are relative to the maximum of the respective asymmetric lenses, with the indication that the geometries (though not the sizes) are effectively the same in these three cases.

lagoonward "dip" of sea level or lagoonward decrease in K. A different pattern of nonuniform recharge would result if there was an interior pond that led to net negative interior recharge because of evapotranspiration losses from ground water. Depending on the strength of the sink and the distribution of the positive recharge, the configuration of the lens could be highly skewed, or, in the extreme, pinched to a local minimum in the interior region. In either case, the effects of the pond would outweigh the effects of the difference in effective sea level.

The two-sea level lens shown in Figure 7 assumes a large "sea-level tilt". The actual on-the-island amount depends on the size of the island, because HLND is a function of the size of the lens in the corresponding one-sea level island. From Figure 6 and the R/K values of islands that are cited there, it would seem that the HLND = 2 of Figure 7 would correspond to a tilt on the island (h_L/L) of 10^{-3}, or 1 m per 1000 m of island width.

The conclusion to be drawn from Figure 7, therefore, is this: while it is true that a sea-level inequality can cause an asymmetry of the lens in a long (*e.g.*, barrier) island, the effect of the sea-level inequality will probably be overshadowed by the effects of nonuniform K and nonuniform R. The relative importance of the sea-level inequality will decrease as the island becomes larger, more highly recharged, or less permeable. With each of these changes, the centerline water-table elevation of the comparison island increases, so a larger "sea-level tilt" (h_L/L) is required to produce the same amount of asymmetry.

It should be pointed out that all these comments on the configuration of the lens assume that the distribution of fresh and salty ground water is equilibrated to the present-day environment of sea level and recharge. This may not be the case in some barrier islands, where there may be some unflushed marine waters dating from earlier submergence. Such waters might occur, for example, in the finer-grained portions of comparatively recently formed members of the class of barrier islands (OTVOS, 1970; SCHWARTZ, 1971) that formed by upward growth of a subtidal shoal.

CONCLUSIONS

(1). If effective sea level is higher on one side of the island than on the other, the lens in the island will be asymmetric because of the difference in position of the outflow. The base of the lens will be skewed toward the low-sea level side of the island (the lagoon side in barrier islands), and the water-table divide will occur closer to the ocean side of the island. The geometry of such a lens can be calculated from a one-dimensional finite-difference model that takes account of the nonzero head in the salt water beneath the lens. Use of an approximate analytical solution, which ignores the nonzero salt-water heads, can result in large errors, especially with respect to the volume of the lens.

(2). In most islands, the effect of the unequal sea levels will probably be outweighed by other factors, namely nonuniform hydraulic conductivity and nonuniform recharge. These other factors become relatively more important as the lens becomes larger because of island width, overall recharge, or overall average hydraulic conductivity. That is, in the larger lenses, the difference in sea level has to become unreasonably large in order to match the effects of relatively small, easily imagined, across-island variations in hydraulic conductivity or recharge.

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