SINKING IN QUICKSAND:
An Applied Approach to the Archimedes Principle

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Popular culture would have people believe that if you fall into quicksand then you will definitely sink to below the surface, but is this realistic, and can we prove scientifically what really happens? To scientifically model quicksand, and a person sinking in it, we need to define what quicksand is. Basically, quicksand is a suspension of sand particles—it could also be dirt as well as mineral and vegetable matter—in water. The ratio of the volume of the solid particles in the suspension to the total volume of the suspension is called the volume fraction. It is the volume fraction that determines the average density of the fluid and the buoyancy force that allows an object to float or sink.

The modeling of the quicksand (or suspension), and whether or not a person will sink in it, is based on Archimedes Principle,[1] whereby the buoyancy (upward acting) force is equal to the weight force of the displaced liquid. So, as a person of fixed mass (downward weight force), sinks into the quicksand, the displaced volume of liquid increases, thereby increasing the upward buoyancy force. The maximum buoyancy force is achieved when the person is totally submerged. If at this point the downward weight force is more than the maximum buoyancy force, the person will sink to the bottom. However, if the downward weight force is balanced before the person is totally submerged, they will remain with part of their body above the liquid.

Applications of the Archimedes principle to the floating of solid bodies in liquid[2] and air can be found elsewhere.[3] Here, we will demonstrate the applicability of this principle to a solid-liquid mixture simulated using a fluidized bed.[4] In this way we aim to respond to the question of if a person will sink in quicksand and provide a different educational approach to this 2,200-year-old principle.

THEORY

Let's consider an object of mass, M, with a partially submerged volume, V_{sub}, in a fluid made of a bead-water mixture (quicksand) of density, ρ_F, as shown in Figure 1 (next page).

Applying Archimedes Principle,[1] if the object is at rest then the downward (weight) force of the object is balanced by the upward (buoyancy) force exerted by the fluid. Mathematically, this can be written as:

$$Mg = \rho_F V_{Sub} g \quad (1)$$

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Eq. (2) states that the submerged volume is equal to the mass of the object divided by the density of the fluid.

$$V_{Sub} = \frac{M}{\rho_F} \qquad (2)$$

The density of the fluid is therefore a combination of the densities of both the liquid and solid particles, depending on the relative proportion of both components in the mixture.

Suppose that the fluid (bead-water mixture) has a volume fraction[1] of particles θ_p, then the volume fraction of the liquid is $(1-\theta_p)$, and thus, the sum of the fractions must be equal to 1. Therefore, the average fluid density can be expressed as:

$$\rho_F = \theta_P \rho_P + (1-\theta_P)\rho_L \qquad (3)$$

Substituting Eq. (3) into Eq. (2), leads to:

$$V_{Sub} = \frac{M}{\theta_P \rho_P + (1-\theta_P)\rho_L} \qquad (4)$$

Therefore, knowing the object mass, M, volume fraction of particles, θ_p, and the densities of the particles, ρ_P, and liquid, ρ_L, the submerged volume, V_S, can be determined. If the submerged volume is less than the total volume of the object, then at least part of it will float above the fluid surface—for a person in quicksand hopefully this part will be the head.

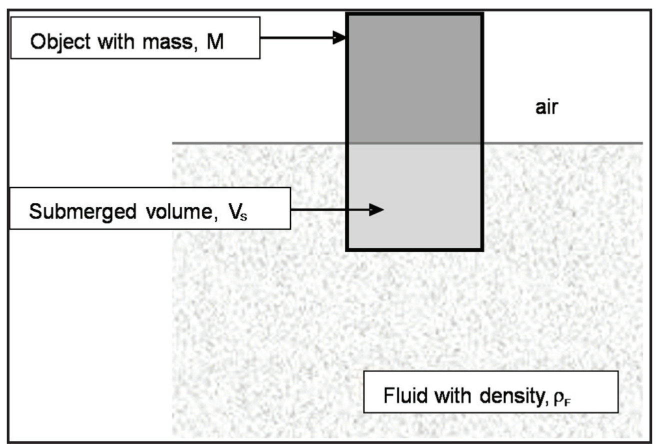

Figure 1. *Object partially submerged in a liquid.*

EXPERIMENTAL

Apparatus

The main part of the experimental program involved setting up a fluidized bed as shown in Figure 2. The apparatus consisted of a cylindrical container (height 9.5 cm, diameter 14.5 cm) sitting on a tripod stand. At the bottom of the container there was a water inlet connected to a tap. At the top of the container was the test tube support and guide that allowed a test tube to move freely up and down in the suspension.

Main Method

- *A known mass of beads was placed inside the cylindrical container.*
- *The test tube holder was then placed on top of the container.*
- *The empty test tube (of known mass) was then placed in the test tube holder.*
- *The tap was turned on (to a set flow rate) to fluidize the bed.*
- *When the bed was fully fluidized, the depth of submersion of the test tube was recorded by reading the graduated scale.*
- *The tap was turned off.*
- *The test tube was removed and filled with a known mass of water.*
- *The test tube was returned to its holder, the bed was re-fluidized at the same flow rate, and the submersion depth was recorded.*
- *The procedure was repeated for different amounts of water in the test tube to measure the effect of changing the mass of the object.*
- *The whole process outlined above was repeated using different volumes of beads inside the fluidized bed, in order to measure the effect of changing the volume fraction, and hence the density, of the fluid.*
- *The procedure was repeated for both plastic and glass beads of differing densities.*

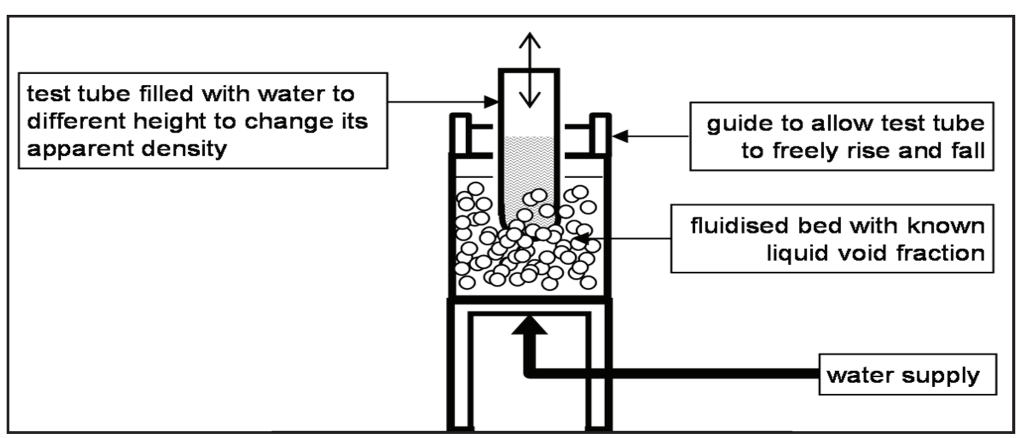

Figure 2. *Experimental apparatus.*

To avoid beads escaping during fluidization so that the volume fraction and density were constant throughout the entire experiment, a thin piece of metal gauze (see Figure 3) was placed over the top of the container. Then, the test tube guide and support were placed on top of the gauze.

1 Defined as the ratio of the mass of the particles to the total mass of particles and liquid present.

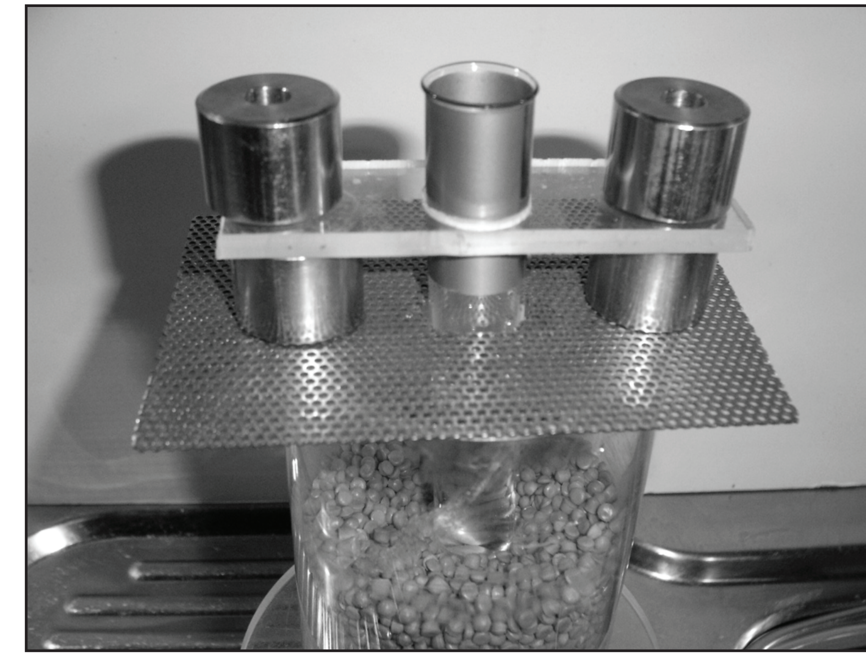

Figure 3. *Gauze cover to prevent overflow of beads*

Determination of the density of water

The density of water was determined by using a balance (able to measure to ±0.001 grams) to obtain the mass of a known volume of water inside a measuring cylinder. At a water temperature of 18 °C, the mass of 100.0 cm^3 of water was 99.300 grams, giving a density, ρ_L, of 0.993 g/cm^3. This compared favorably with a reported value of 0.999 g/cm^3.[5]

Determination of the density of beads

The analysis carried out to determine the average density of the plastic and glass bead can be described as followed:

- *A quantity of beads was placed in a measuring cylinder, on a balance so we could simultaneously determine mass and volumes. The mass of the beads was recorded.*
- *A known amount of water (M_W, V_W) was then added to the measuring cylinder to a known volume, V_T. The volume and mass of the beads plus the water was recorded.*
- *The density of the beads (solids), ρ_P, was calculated as a function of the density of water (0.993 g/cm^3 from above); mass of beads, M_p; total volume of water and beads, V_T; and mass of water added, M_L. This calculation is described below.*

The density of the beads can be written as:

$$\rho_P = \frac{M_P}{V_p} \qquad (5)$$

where the particle volume can be expressed as a function of the total measured volume, V_T, and the volume of added water, V_W.

$$V_p = V_T - V_W \qquad (6)$$

Expressing the volume of water as a function of its mass and density we rewrite Eq. (6) as

$$V_p = V_T - \frac{M_W}{\rho_W} \qquad (7)$$

Introducing Eq. (7) into Eq. (5) we obtain

$$\rho_p = \frac{M_p}{V_T - \left(\frac{M_L}{\rho_L}\right)} \qquad (8)$$

Using the above equations, the calculations are:

$$\text{Plastic Beads:} \quad \rho_p = \frac{M_p}{V_T - \left(\frac{M_L}{\rho_L}\right)} = \frac{34.630}{92 - \left(\frac{62.193}{0.993}\right)} = 1.179\,g/cm^3 \qquad (9)$$

$$\text{Glass Beads:} \quad \rho_p = \frac{M_p}{V_T - \left(\frac{M_L}{\rho_L}\right)} = \frac{58.793}{70 - \left(\frac{45.587}{0.993}\right)} = 2.441\,g/cm^3 \qquad (10)$$

While it was not possible to compare the measured density of the plastic beads with published values—because there is a huge range of plastic densities—the density of the glass beads fell within the reported range of 2.4-2.8 g/cm^3.[6]

Determination of the volume of the test tube

The volume of the test tube was calculated in two parts—the cylindrical stem of the test tube and the dished bottom of the test tube. Using a ruler and calculator, necessary measurements were taken and substituted into the equations to calculate the volume of the test tube.

To measure the volume of the dished bottom of the test tube, a measuring cylinder was filled to the brim with water, and then placed on the balance and zeroed. The dished bottom was immersed into the measuring cylinder, allowing the displaced water to spill out. Then the test tube was removed and the remaining liquid in the measuring cylinder was weighed again. The difference in the mass of water before and after the test tube was immersed gives the mass of the displaced liquid.

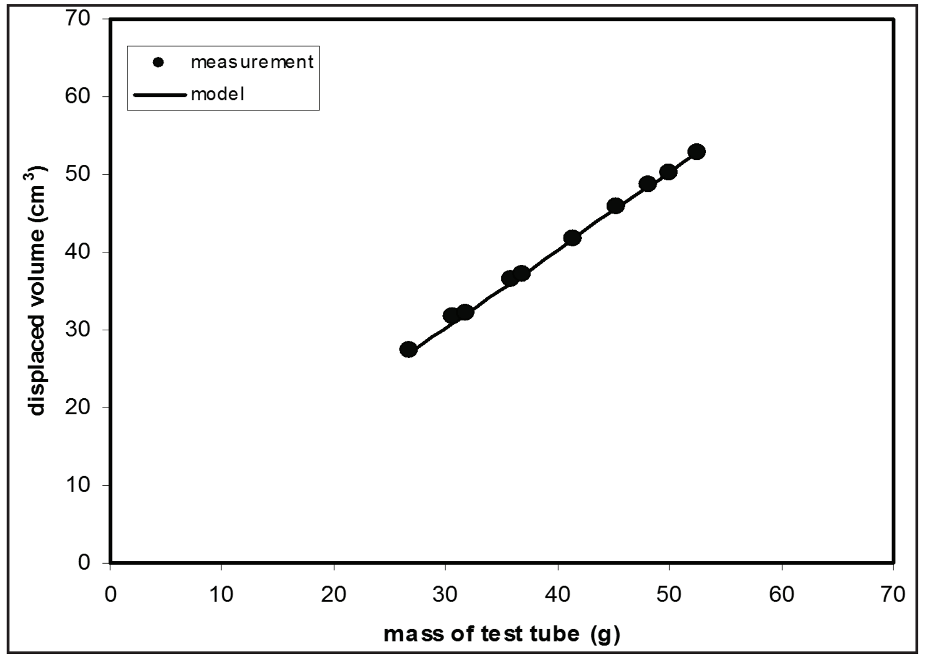

***Figure 4**. Displaced volume versus test tube mass (water only).*

Risk analysis

The experiment involved running water, a glass test tube and measuring cylinder, plastic and glass beads, and an electronic balance. With the balance there was a chance of electric shock if the appliance came into contact with water. Steps were taken to avoid this. With the apparatus itself, the water hose was firmly attached to the tap and the experiment was performed in the sink so that any spills were contained. With the glassware, care was taken to avoid breakage. However, the top of the test tube did break due to constant handling that the experimental procedure required. There was no bodily harm because the test tube was being handled with a cloth. The beads were handled with care to avoid spillages on the floor and creating a hazardous area to slip. During operation the only hazard was flowing water, which was minor due to a low water velocity. Safety glasses were not available.

RESULTS AND DISCUSSIONS

Control experiment: water-only run

In the first instance the first experimental run was performed without any beads in the fluidized bed. This run could be considered to be the control experiment, in that the experimental system is demonstrated to confirm Archimedes Principle, when the density of the fluid is just that of water.

Figure 4 shows the submerged or displaced volume plotted as a function of the mass of the test tube which was filled with only water. The experimental results are shown as closed circles, while the model prediction, from Eq. (2), is given as a solid line. It can be seen that the displaced volume increases as the mass of the test tube increases. There is good agreement between the data and model, adding confidence to the experimental methodology.

Fluidized bed: use of plastic beads

Figures 5 and 6 (following pages) contain the results for the submerged volume as a function of the mass of the test tube at plastic bead volume fractions of 0.25 and 0.34, respectively. At a volume fraction of 0.25 (Figure 5) repeat experiments were performed to test the reproducibility of the measurements. It can be seen that there was good reproducibility between the two runs. At low test tube mass, the model line matched the measurements quite well. However, at higher test tube masses, the model over-predicted the displaced volume. The reason for this was thought to be due to the upward motion of the water adding an extra upward force that resulted in a lower displaced volume. While the additional "fluid motion" force was not accounted for in the model, it only became significant at higher water flow rates. Care was taken to ensure that the fluid flow was held constant throughout each of the rest of the runs.

Fluidized bed: use of glass beads

The plastic beads were substituted with glass beads and the experiments repeated for different volume fractions. The results are given in Figures 7 and 8 (following pages) for volume fractions of 0.11 and 0.23, respectively. In Figure 7 a different trend was observed, in that the measured values were larger than the model predictions. This was because the liquid flow rate was insufficient to fully fluidize the bed, resulting in a lower average density and upward buoyancy force on the test tube. The water flow rate was increased for the higher volume fraction to ensure all particles were fluidized. The model and measurements agreed very well.

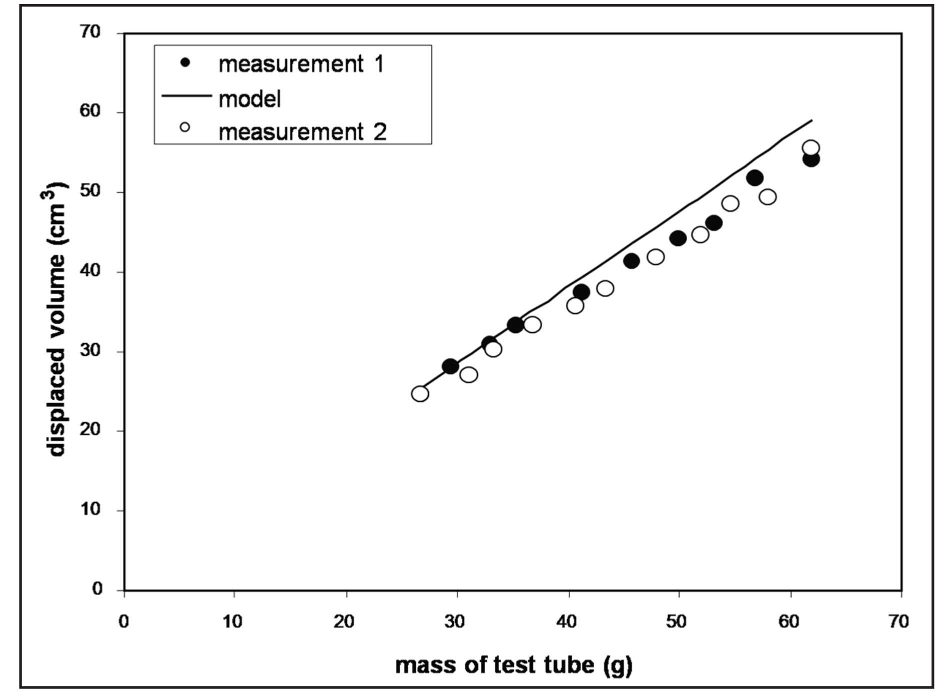

***Figure 5.** Displaced volume versus mass (Plastic Beads, θ_S =0.25).*

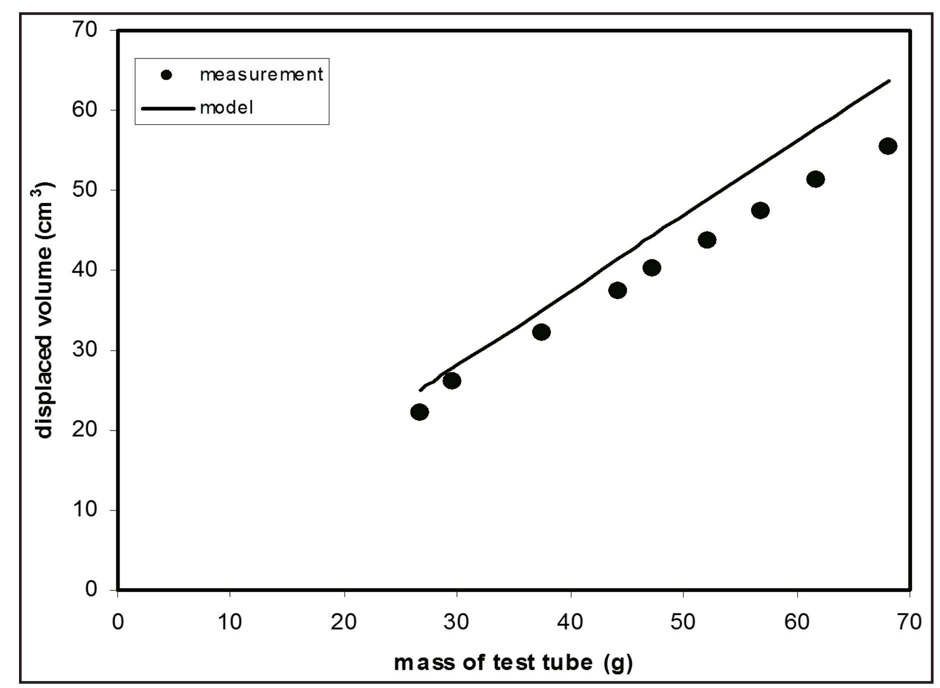

***Figure 6.** Displaced volume versus mass (Plastic Beads, θ_S =0.34).*

It is interesting to note that the upward thrust of the flowing liquid had much less effect for the glass beads when compared with that for the plastic beads. This was thought to be due to the more even distribution from the water exit at the bottom of the container, due to the smaller size and greater density of the glass beads.

Comments on errors

The main errors in the experimental methodology were due to measuring the submersion depth and ensuring all of the bed was fluidized. It was often hard to measure the submersion depth of the test tube, especially at high volume fractions of beads and upward water flow rate. The uncertainty in the measurement was estimated to be ±4mm.

In addition to the "experimental" errors, there was also a limitation, or error, in the model used. It did not include an additional upward force due to the water flow. Consequently, in a number of cases the measurements deviated from the model curve. This extra force would need to be modeled using the known liquid flow rate through the fluidized bed. Unfortunately, in the current experimental setup the flow rate could not be measured easily.

In relation to the upward flow of liquid, in real quicksand there do not appear to be high liquid velocities. Under these conditions, Archimedes Principle without modification to account for liquid velocity but accounting for the effect of solids on the fluid density would appear to model the system very well.

Application of the theory to a person in quicksand

The previous results have highlighted that the model can be used to predict the displaced volume provided that the average density of the fluid (solids plus liquid) is taken into account. In order to apply Eq. (4) to the case of a person sinking into quicksand it is best to substitute the mass of the person by their density, ρ, and total volume, V. The equation becomes:

$$V_{Sub} = \frac{M}{\theta_p\rho_p + (1-\theta_p)\rho_L} = \frac{\rho V}{\theta_p\rho_p + (1-\theta_p)\rho_L} \qquad (11)$$

$$\frac{V_{Sub}}{V} = \frac{\rho}{\theta_p\rho_p + (1-\theta_p)\rho_L} \qquad (12)$$

where V_{Sub}/V is the fraction of the total volume of the person that is submerged.

Given that the density of a person ranges between 1.01-1.07 g/cm^3 [7] and the density of sand (quartz) is 2.65 g/cm^3,[8] the submerged volume fraction can be plotted as a function of the solids volume fraction. The results are given in Figure 9 (next page) for a person of density 1.07 g/cm^3. However, it is likely that the person will carry other objects for maintenance that would increase his or her relative density. In some cases fully packed rucksacks containing cans of food, or metal objects such as flashlights or lamps, would largely increase the person's density beyond the normal values.[7] In these cases, which are not uncommon, the person is more likely to sink. Thus, Figure 9 also considers the case of a slightly higher human density that could incorporate the effect of the goods carried when falling into quicksand.

Figure 9 shows that the person will be totally submerged when the volume fractions of the quicksand are 0.05 and 0.16 for densities of 1.07 and 1.25 g/cm^3, respectively. These values correspond to the condition at which a person is neutrally buoyant, in other words, the density of the fluid (quicksand) is equal to the density of the person. At higher volume fractions of solids in the quicksand the person will start to float. For quicksand, the solid volume fraction would be at least 0.5, and from the graph at least 40 or 30 percent of the person's volume is likely to remain above the liquid surface depending on the person's density (Figure 9). The analysis is for a maximum person density. Less dense people will be even more likely to float in the quicksand.

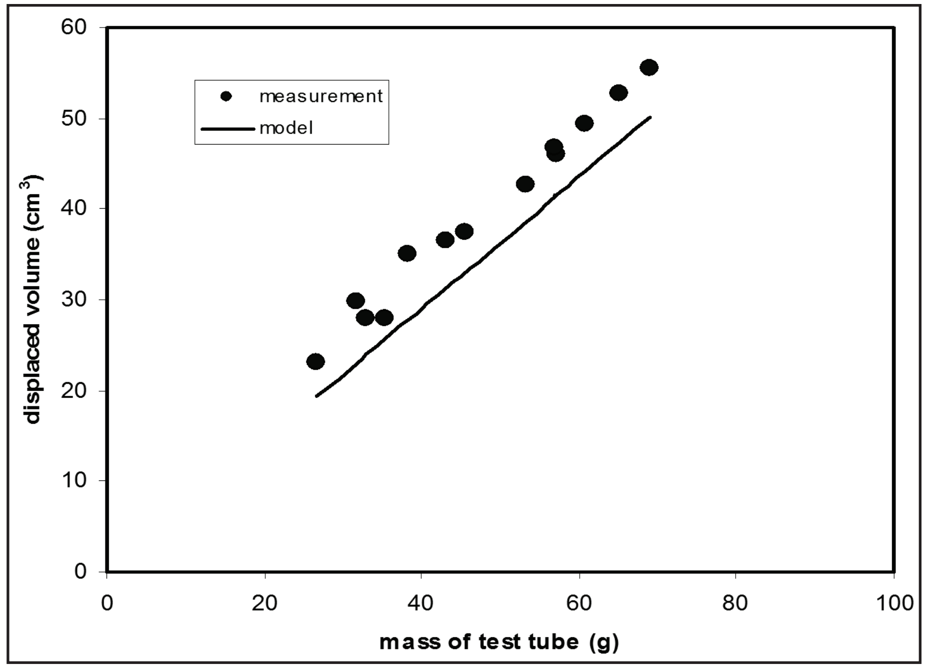

Figure 7. *Displaced Volume versus Mass (Glass Beads, θ_s =0.11).*

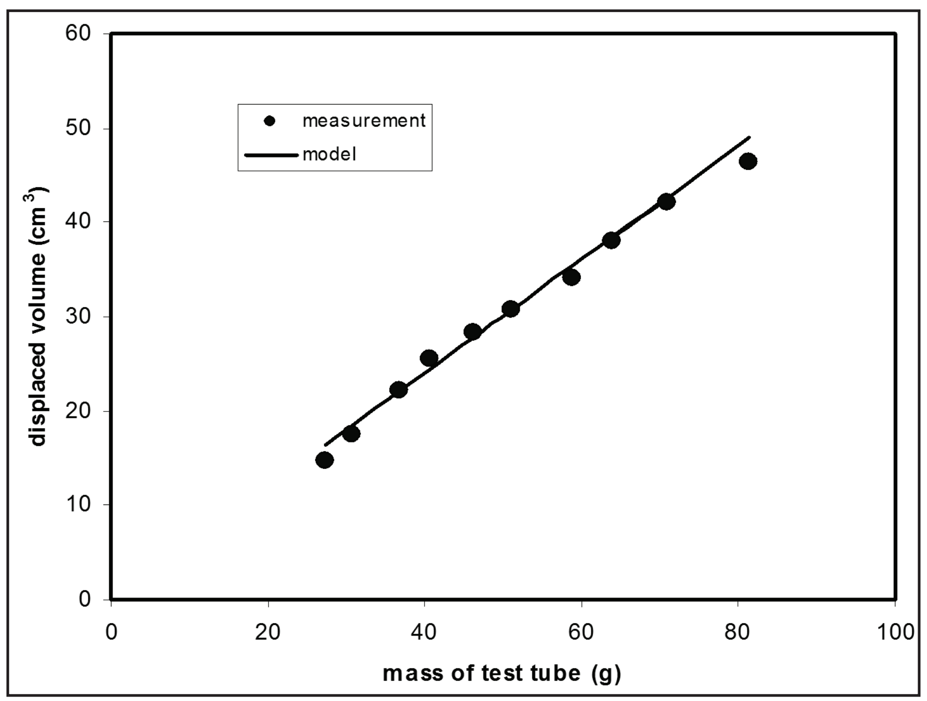

Figure 8. *Displaced Volume versus Mass (Glass Beads, θ_s =0.23).*

Furthermore, the analysis reported here only considers the case of a still object within the fluidized bed. When we consider a real situation of a person, she or he is likely to move, producing an expansion of the surrounding sand. Such expansion is associated to a local decrease in the packing of the surrounding areas and thus of the local volume fraction of the bed. Thus, the motion would change the mechanical balance and the person would continue sinking. This situation is somehow similar to that of the vibration of the bed or, in nature, to an earthquake. In these situations, the energy of the mechanical waves produces the fluidization and thus the expansion of the material with a considerable decrease in the volume fraction of the bed with the final consequence of the sinking of the object.

Finally, it is important to notice that bubbles may also be present in quicksand. These bubbles will contribute as an extra term in the denominators of Eqs. (3), (11), and (12). This extra term is proportional to the bubble density (density of air) and the volume fraction of bubbles. Due to the fact that the air density is smaller than the density of liquid or particles, the denominator of the above mentioned equations will decrease making the fraction of submerged volume of the person increase (Eq. 12).

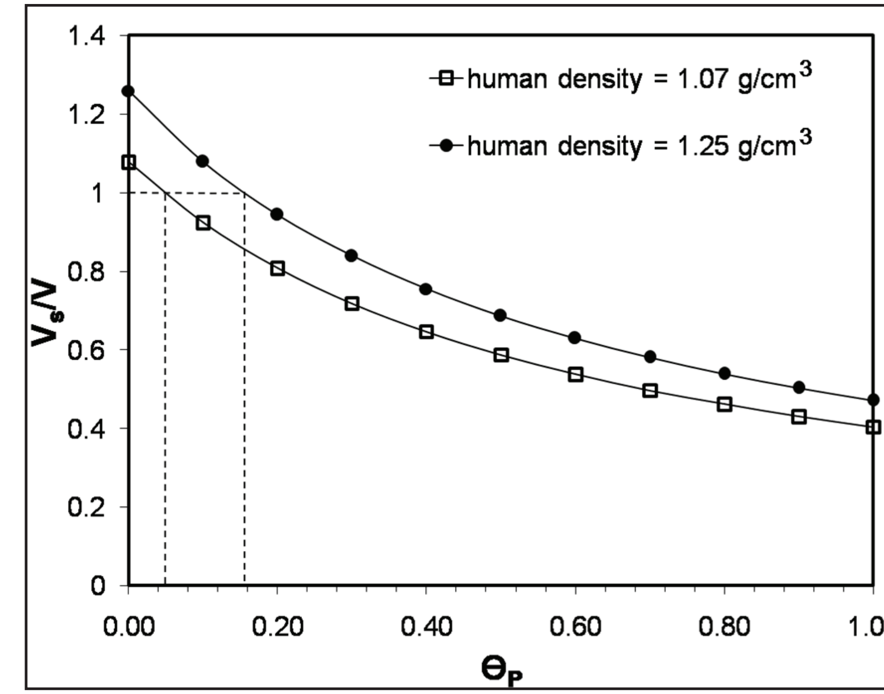

Figure 9. *Fraction of volume submerged versus volume fraction of the sand.*

CONCLUSION

The results confirmed that the Archimedes principle could be applied to the fluidized bed as long as the mean density of the fluid (liquid and solids) was taken into account. The extent to which a person would sink to the bottom in quicksand was therefore found to depend on both the density and volume fraction of the solid particles in the bed. As the density and volume fraction of the solid particles in the quicksand increased, it would be less likely that a person would sink. To a lesser extent the density of the person would influence how far they would sink, but even for the densest person, they are unlikely to become fully submerged.

ACKNOWLEDGMENTS

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NOMENCLATURE

- g gravity acceleration [9.8 m /s^2]
- M Mass of the object [kg]
- M_P Total mass of the beads used in the experiment [kg]
- V Total volume of the object [kg]
- V_{sub} Volume of the object that is submerged in the bead-water mixture [m^3]
- V_P Total volume of beads used in the experiment [m^3]
- V_W Total volume of water used in the experiment [m^3]
- V_T Total volume of the bead-water mixture m^3
- ρ_F Fluid density (bead-water mixture) [kg/m^3]
- ρ_P Particle density [kg/m^3]
- ρ_L Liquid density [kg/m^3]
- θ_P Volume fraction of particles in the bead-water mixture [-]

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