ChE classroom

A USEFUL FORMULA IN PROCESS CONTROL

W. J. CHEN
Ohio University
Athens, OH 45701

A NON-INTERACTING THIRD order system plays an important role in process control as a typical system employed in the classroom and laboratory to illustrate some of the control concepts. It is the simplest system that can be used to show instability in the closed loop control with any of the three controllers (proportional, proportional-integral, proportional-integral-derivative). Lower order systems do not exhibit instability with proportional control. It is also used to demonstrate some of the control methods, such as root-locus, frequency-response, and loop tuning. In all these methods, the parameters to be determined are the maximum overall gain, K_{max}, and the critical frequency, w_c. When the overall gain is greater than K_{max}, the control system is unstable. Therefore K_{max} is probably the most important parameter in control design. For this reason, a lot of time and effort can be saved if a simple formula can be devised for K_{max}.

Harriot [1] developed a formula to calculate K_{max} and w_c, but it was not widely used. Routh's array [2] was most widely used, but it was not simple. My students and I have employed a very simple formula which is based on the fundamental information of the system, namely the time constants. If the three time constants are T_1, T_2 and T_3, the maximum gain can be calculated directly by

$$K_{max} = \sqrt{1 + (w_cT_1)^2}\ \sqrt{1 + (w_cT_2)^2}\ \sqrt{1 + (w_cT_3)^2} \qquad (1)$$

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Wen-Jia R. Chen obtained his BSChE from National Taiwan University, and his MS and PhD from Syracuse University. He worked at Allied Chemical Corporation and the University of New Mexico before joining Ohio University. His current research interests are in coal conversion and process control. He is also planning to integrate interactive graphics with ASPEN.

where

$$w_c = \sqrt{(T_1 + T_2 + T_3)/(T_1T_2T_3)} \qquad (2)$$

The above equations can be derived from the Bode stability criterion.

$$\tan^{-1}(w_cT_1) + \tan^{-1}(w_cT_2) + \tan^{-1}(w_cT_3) = 180 \qquad (3)$$

$$K_{max} \cdot \frac{1}{\sqrt{1+(w_cT_1)^2}}\ \frac{1}{\sqrt{1+(w_cT_2)^2}}\ \frac{1}{\sqrt{1+(w_cT_3)^2}} = 1 \qquad (4)$$

Take the tangent of Eq. (3) and expand

$$w_cT_1 + w_cT_2 + w_cT_3 - w_cT_1w_cT_2w_cT_3 = 0 \qquad (5)$$

Therefore

$$w_c = \sqrt{(T_1 + T_2 + T_3)/(T_1T_2T_3)} \qquad (2)$$

And from Eq. (4)

$$K_{max} = \sqrt{1 + (w_cT_1)^2}\ \sqrt{1 + (w_cT_2)^2}\ \sqrt{1 + (w_cT_3)^2} \qquad (1)$$

Some interesting information can be deduced from Eq. (1). First, the maximum gain depends only on the relative magnitude of the time constants. If the dominant time constant is T_1, and the other two time constants, T_2 and T_3, are expressed in terms of T_1

$$T_2 = X\,T_1, \qquad T_3 = Y\,T_1 \tag{6}$$

then Eq. (1) becomes

$$K_{max} = \sqrt{1 + \frac{1+X+Y}{XY}}\ \sqrt{1 + \frac{1+X+Y}{Y}X}\ \sqrt{1 + \frac{1+X+Y}{X}Y} \tag{7}$$

which depends only on the ratios of the time constants. Second, the maximum gain is the same for two systems with the time constants as the reciprocals of each other. In other words, if 1, 1/X, 1/Y are substituted into Eq (1), Eq. (7) remains the same. Third, the minimum K_{max} happens when all three time constants are identical. The corresponding value is 8. Finally, K_{max} goes to infinity when any one of the ratios goes to infinity which corresponds to a lower order system, and hence is always stable.

REFERENCES

1. Harriot, P., *Process Control*, McGraw-Hill, New York, 1964
2. Routh, F. J., *Dynamics of a System of Rigid Bodies*, 3rd ed., Macmillan & Co., Ltd., London, 1877 ☐

REVIEW: Liquids

Continued from page 77.

modynamic properties have been made, including new methods for studying and understanding fluid behavior near critical and tricritical points. On the theoretical side, the 70's and 80's have seen the rapid development of computer simulation methods and perturbation theory, which have placed the interpretation of liquid properties on a much sounder footing. In particular, the use of simulation data to test theories has eliminated the need to guess the intermolecular forces—a serious source of error in almost all of the tests of theories prior to 1970.

The aim and general organization of the book are the same as for the previous edition. The first two chapters give an introduction to liquid properties and their classical thermodynamics, and are similar to those of the last edition. The next chapter covers fluid behavior near the critical point, and has been substantially updated to cover recent developments. The next three chapters cover our experimental knowledge of thermodynamic properties and phase equilibria for liquid mixtures at both low and high pressures. The coverage is similar to that in the last edition, but the material has been reorganized and updated. Welcome additions include the introduction of phase diagrams plotted in terms of field variables (temperature T, pressure p, and chemical potential μ in place of T, p, and x, where x is the mole fraction), a section on tricritical points, and the use of the classification scheme of van Konynenburg and Scott in discussing the behavior of fluid mixtures at high pressure. The last two chapters of the book cover the statistical thermodynamics of liquids and liquid mixtures. Much has been added here, particularly on the recent developments on fluids of nonspherical molecules, perturbation theory, and measurements of molecular correlation functions. The chapter on intermolecular forces, present in the last edition, has been removed, although there is a brief discussion on this subject in Chapter 7. The authors conclude that the statistical thermodynamics of mixtures of rigid (*i.e.* non-flexible) neutral molecules has made major advances and is now on a wound footing. The main barrier to progress remains our relative ignorance of intermolecular forces for all but the simplest molecules.

This latest edition of the book will be indispensable to those working on liquid properties. ☐

A PICTORIAL APPROACH TO MOLECULAR STRUCTURE AND REACTIVITY

by R. F. Hout, Jr., W. J. Pietro, and W. J. Hehre
Published by Wiley-Interscience,
Somerset NJ 08873 (1984),
403 pages, $39.95

Reviewed by
Gar B. Hoflund
University of Florida

It is well known that molecular orbital theory and relatively simple symmetry arguments can provide important information about the electronic and geometrical structure of molecules and their chemical reactivity. In this approach, extended molecular orbitals are constructed from hydrogen-like atomic arbital. While the visualization of atomic orbitals is quite easy, the visualization of molecular orbitals can be quite difficult even for relatively simple molecules. This book provides a pictorial representation of high-lying (valence) molecular orbitals for a large number of polyatomic inorganic, organic, and organometallic molecules. The molecular orbitals have been generated using the GAUSSIAN-83 computer program, and in this book photographs taken of a high rsolution graphical display of the molecular orbitals are presented. In addition to the filled levels, the lowest-unfilled molecular orbital (LUMO) is also shown for each molecule. This book provides an important source of information for those trying to understand molecular interactions on a fundamental level. ☐