INTRODUCING THE CONCEPT OF FILM HEAT TRANSFER COEFFICIENTS

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STUDENTS OFTEN STRUGGLE to gain an appreciation of the concept of a heat transfer coefficient even though they are familiar with the concept of thermal conductivity. An example of heat loss from single and double glazed windows (which is developed later in this paper) helps to bridge this divide; a beneficial link with familiar surroundings is established.

BACKGROUND KNOWLEDGE

Students should already be familiar with the method for calculating heat flow along a lagged bar, as shown in Figure 1. This involves a straightforward application of the following equation (which is often called Fourier's law)

$$q = -kA\left(\frac{d\theta}{dx}\right) \tag{1}$$

It is also necessary that the concept of interfacial temperature be understood. This may be introduced via the composite slab problem, which is both interesting and relevant. In this problem it is supposed that there are two slabs of equal area A, of thickness t_1 and t_2, and with thermal conductivity k_1 and k_2, respectively. Let the temperatures be defined by Figure 2. Now the flow of heat through each slab is the same; therefore

$$q = \frac{k_1 A(\theta_{hot} - \theta_i)}{t_1} = \frac{k_2 A(\theta_i - \theta_{cold})}{t_2} \tag{2}$$

The interfacial temperature will rarely be known, but there are two equations, and q and θ_i are generally the unknowns. Rearrangement gives

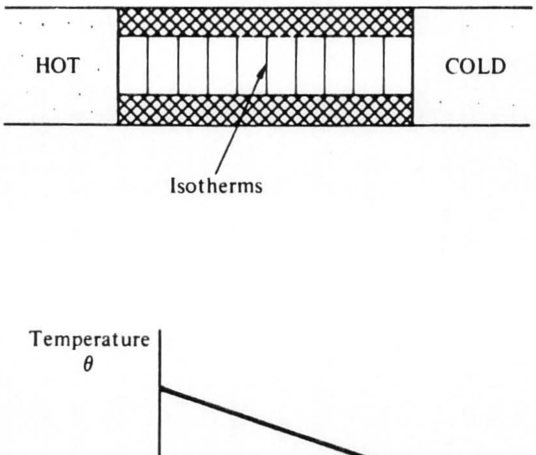
$$\theta_{hot} - \theta_i = q\frac{t_1}{k_1 A} \tag{3}$$

and

$$\theta_i - \theta_{cold} = q\frac{t_2}{k_2 A} \tag{4}$$

Addition of Eqs. (3) and (4) gives

$$\theta_{hot} - \theta_{cold} = q\left(\frac{t_1}{k_1 A} + \frac{t_2}{k_2 A}\right) \tag{5}$$



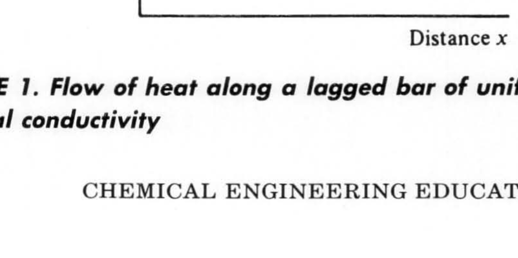
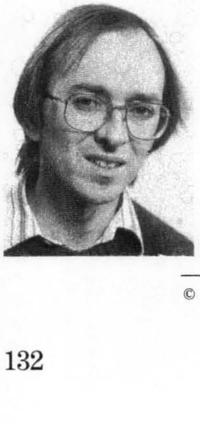


FIGURE 1. Flow of heat along a lagged bar of uniform thermal conductivity



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that is

$$q = \frac{\theta_{hot} - \theta_{cold}}{\left(\frac{t_1}{k_1 A} + \frac{t_2}{k_2 A}\right)} = \frac{(\theta_{hot} - \theta_{cold})A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2}\right)} \qquad (6)$$

This is similar to Ohm's law: q is the flow of heat instead of current; $\theta_{hot}-\theta_{cold}$ is equivalent to the driving force, the potential difference; and the t/kA terms are thermal resistances. The equation can be generalized to give the heat flow through a composite of many layers

$$q = \frac{(\theta_{hot} - \theta_{cold})A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3} + \dots\right)} \qquad (7)$$

where $\theta_{hot}-\theta_{cold}$ are the temperatures of the outer surfaces of the composite.

HEAT LOSS ACROSS WINDOWS

An Oversimplification

A familiar example is the loss of heat through closed windows. Students can be encouraged to estimate the loss using the above theory. For illustrative purposes, single and double glazed windows of the following specifications will be assumed: single glazed 4 mm thick glass with k = 1.05 $Wm^{-1}K^{-1}$; double glazed

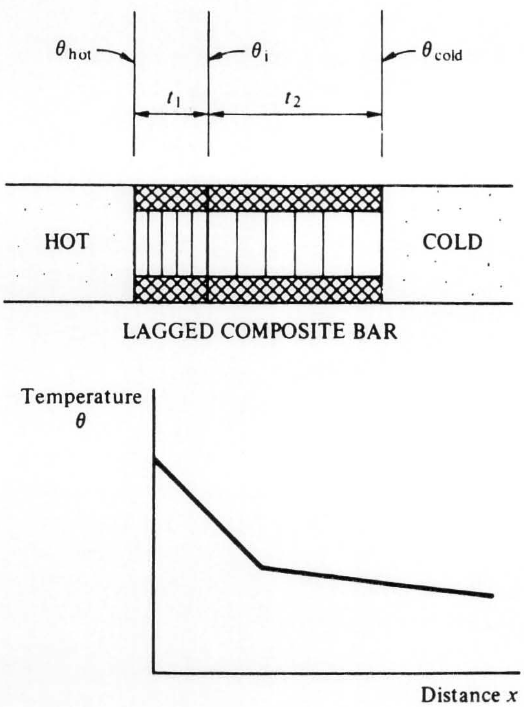

FIGURE 2. Flow of heat along a composite bar (and definition of temperatures used in text)

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units incorporating two panes of 4 mm thick glass and a 12 mm air gap whose thermal conductivity is taken to be 0.023 $Wm^{-1}K^{-1}$. The area of glazing will be taken as $3m^2$, the room temperature as 20°C, and the air temperature as –4°C.

It could be argued that for the calculations the temperatures should be in Kelvin, not degrees Celsius. However, the numerical results are not affected since temperature differences are the same in K and °C. This is an opportunity for pointing out that normal engineering practice does not slavishly follow the SI set of units and °C will be retained. Application of Eq. (7) leads to the following estimates:

$$\text{heat loss through double glazing} = \frac{[20-(-4)]3}{\left[\frac{0.004}{1.05} + \frac{0.012}{0.023} + \frac{0.004}{1.05}\right]} = 136\text{ W}$$

$$\text{heat loss through single glazing} = \frac{[20-(-4)]3}{\frac{0.004}{1.05}} = 18{,}900\text{ W}$$

The last figure is clearly excessive since 18.9kW is greater than the heat input for a whole house! If the inside surface of the pane were 20°C and the outer surface –4°C, then the heat loss would undoubtedly be in excess of 18kW. It is interesting to ask students if the temperature gradients shown in Figure 3 are reasonable.

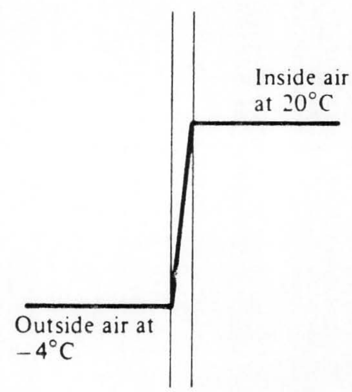

FIGURE 3. Temperature profile across a pane of glass in the absence of boundary layers

At this juncture, an opportunity arises to point out that one must be explicit about one's assumptions. Figure 3 implies that the outside air which is close to, and right up to, the window is all at –4°C, despite a large outflow of heat. Similarly, there is no temperature gradient on the room side. The model which was implicitly assumed, and which has been made explicit in Figure 3, is unrealistic. The model illustrated in Figure 4 can be introduced as being much more realistic, but still not exact. An engineer learns the importance of using intelligent approximations and of making the best possible estimate from incomplete information. In a small way this is illustrated by the current problem.

Having intuitively noted that there are regions close to both glass-air boundaries over which the temperature changes from bulk air temperature to glass temperature, it is useful to introduce a physical picture so that calculations can be performed. It may be agreed that a reasonable approximation is to assume that the air, both on the inside and the outside, can be represented by a near stagnant film or boundary layer across which there is an appreciable temperature change and a well mixed bulk which is isothermal.

It is reasonable to assume that the film thicknesses would be 2mm for the room side and 1.5mm for the outside, if the wind speed is low. A reduction to 1mm is appropriate if the wind speed is higher. Their recalculations should give the following results:

$$\text{heat loss through single glazing (low wind speed)} = \frac{[20-(-4)]3}{\left(\frac{0.002}{0.023}+\frac{0.004}{1.05}+\frac{0.0015}{0.023}\right)} = 462\text{ W}$$

$$\text{heat loss through single glazing (high wind speed)} = \frac{[20-(-4)]3}{\left(\frac{0.002}{0.023}+\frac{0.004}{1.05}+\frac{0.001}{0.023}\right)} = 536\text{ W}$$

The thicknesses and the resulting heat loss values are reasonable, and the model (which is one of pure conduction through a stagnant layer) might be of interest and, in some circumstances, of use. However the teacher will undoubtedly wish to point out that the aim is to have a value for the thermal resistance, and it does not matter if the heat loss mechanism is a combination of convection and conduction, provided an accurate estimate can be made. In the above example, the inside thermal resistance, t/(kA), is 0.002/(0.23 × 3) = 0.029 K W^{-1}. Taking the reciprocal (kA/t) and converting it into per area form (k/t), one has the heat transfer coefficient. In this case it equals 0.023/0.002 = 11.5 $Wm^{-2}K^{-1}$. This example has not only introduced the concept of a heat transfer coefficient but

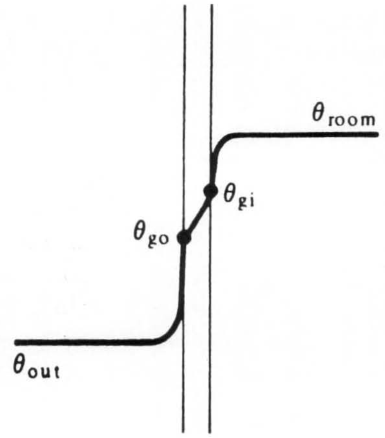

FIGURE 4. Temperature profile across a pane of glass in the presence of boundary layers

also illustrates that a balance between theory and empiricism has been productive. The insights into the physics underpinning heat transfer coefficients lead to a better theoretical understanding. The coefficients that are subsequently developed are not tied to any particular model. They can be treated as purely empirical constants of proportionality, the knowledge of which permits (given knowledge of surface area and temperature difference) the calculation of the amount of heat transferred.

While one can always find a film thickness to give a reasonable result, one can rarely predict the appropriate film thicknesses for a new situation. However, knowledge of the film thicknesses is now seen to be insignificant. In contrast, the important film heat transfer coefficients can readily be calculated from predictive equations. These enable an engineer to give an *a priori* prediction of performance under changed circumstances. The confidence attached to this prediction is enhanced if the predictive equation has some theoretical underpinning.

OVERALL HEAT TRANSFER COEFFICIENT

The above example can be used to introduce the concept of overall heat transfer coefficients. The method for combining these coefficients is similar to the method for combining thermal resistances, and an analogue for Eq. (7) will be obtained. The temperatures for the current example are defined in Figure 4. Remembering that the heat flow through the glass and the two boundary layer films is the same, the students should obtain

$$q = h_{room}A(\theta_{room}-\theta_{gi}) = \frac{k_g A(\theta_{gi}-\theta_{go})}{t_g} = h_{out}A(\theta_{go}-\theta_{out}) \qquad (8)$$

where h_{room} is the heat transfer coefficient for the inside (or roomside) boundary layer, and h_{out} is the heat transfer coefficient for the outside boundary layer.

Rearrangement and addition as before gives

$$(\theta_{room} - \theta_{out}) = \frac{q}{A}\left(\frac{1}{h_{room}} + \frac{t_g}{k_g} + \frac{1}{h_{out}}\right)$$

or

$$q = \frac{(\theta_{room} - \theta_{out})A}{\left(\frac{1}{h_{room}} + \frac{t_g}{k_g} + \frac{1}{h_{out}}\right)} \quad (9)$$

The outside heat transfer coefficient will be dependent on wind speed and window position, which need to be determined, but the exact mode of heat transport (*e.g.*, the balance between convection and conduction) is unimportant and of scientific, not engineering, interest. The rate of flow of heat per unit area per unit temperature difference is

$$\frac{q}{A(\theta_{room} - \theta_{out})}$$

This is, of course, the overall heat transfer coefficient, U, and from the above equation its relationship to the individual coefficients is seen to be of a reciprocal nature

$$\frac{1}{U} = \frac{1}{h_{room}} + \frac{t_g}{k_g} + \frac{1}{h_{out}} \quad (10)$$

It may be pointed out that this is analogous to the summing of electrical resistances; the term on the left-hand side is the overall resistance to heat transfer and those on the right are the individual resistances.

DISCUSSION

It may be noted that the assumption of a stagnant layer of air between the panes of the double glazed units was also an oversimplification. The circulation currents within the enclosed space reduce the insulating effect. In order to reduce this loss of insulating power, certain manufacturers fill the space with inert gases which are several times denser than air. Frame construction also influences heat loss, and the final overall heat transfer coefficients range from 2.0 to 3.5 $Wm^{-2}K^{-1}$ for double glazed units. This compares favorably with the 7 $Wm^{-2}K^{-1}$ of typical single glazed windows, but the difference is not as dramatic as students and others first suppose.

The pedagogic value of the example is not limited to the introduction of the overall heat transfer equation. There is the opportunity to develop (a) students' understanding of natural convection by considering in greater detail the physical process occurring in the enclosed cavity between the panes, and (b) their appreciation of forced convection by considering the effect of wind speed upon the outside film heat transfer coefficient.

ACKNOWLEDGEMENT

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NOMENCLATURE

A	area
h	film heat transfer coefficient
k	thermal conductivity
q	heat flux
t	thickness
U	overall heat transfer coefficient
x	distance
θ	temperature

Subscripts

1,2	refers to slabs as shown in Figure 2
cold,hot,i	refers to cold-side, hot-side, and interfacial positions as shown in Figure 2
g	glass
gi	glass-room interface
go	glass-outside interface
out	outside
room	room-side ❒

ChE book reviews

INTRODUCTION TO PHYSICAL POLYMER SCIENCE

by L.H. Sperling

John Wiley & Sons, One Wiley Dr., Somerset, NJ 08873; $39.50 (1986)

Reviewed by
F. Rodriguez
Cornell University

This textbook is written at the level of the senior or beginning graduate student who has had no previous courses in polymers. It is presumed that a course in organic polymer chemistry will follow.

Recognition of the importance of polymers for chemists and chemical engineers has yet to be acknowledged in many departments. However, the

Continued on page 172.