IS MATTER CONVERTED TO ENERGY IN REACTIONS?

Paul K. Andersen
New Mexico State University • Las Cruces, NM 88003

Perhaps the most famous equation of physics is Einstein's mass-energy relation

$$E = mc^2 \tag{1}$$

Although this equation is well known, it is often misunderstood to mean that matter is converted to energy (or vice versa) in reactions; that matter is a form of energy; and that the principle of energy conservation must be modified. Consider, for example, the following excerpt from a general chemistry text:[1]

> Regardless of the classification used—physical reaction, phase change, ordinary chemical change, chemical change, nuclear reaction—changes in matter involve *the change of matter to energy* if the reaction *evolves energy,* and *the change of energy to matter* if the reaction *absorbs energy.* Energy and matter are thus interchangeable. The scope of the conservation principle is therefore enlarged to include energy as a form of matter or matter as a form of energy.... The convertibility of matter and energy is described by the equation $E = mc^2$, predicted by Albert Einstein in 1905....

As we shall see, Eq. (1) does not say that matter and energy are interchangeable, or that matter is a form of energy. Nor does it extend the principle of conservation of energy.

MATTER INTO ENERGY?

One difficulty with the foregoing quote is an ambiguity over the meaning of the word *matter.* There are at least two common ways to measure the quantity of matter in a body: by its mass, or by the numbers of elementary particles it contains. The latter is usually expressed in *moles.*

It is easy to show that the constituents of matter are not created or destroyed in ordinary chemical reactions. For example, hydrogen and oxygen react according to the equation

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

On the left side of the equation we find two moles of hydrogen and one mole of oxygen; the same is true of the other side. There is no conversion of matter into energy, or vice versa.

What about nuclear reactions? Consider one of the fission reactions that may occur when a neutron is absorbed by uranium-235:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{142}_{56}Ba + ^{91}_{36}Kr + 3\,^{1}_{0}n \tag{2}$$

A count of the protons, electrons, and neutrons before and after the reaction shows no change:

Before	After
92 protons	92 protons
92 electrons	92 electrons
144 neutrons	144 neutrons

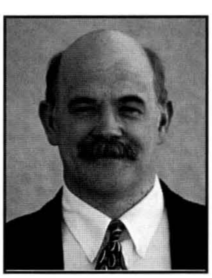
Once again we have an example in which the constituents of matter are conserved in an exothermic reaction. There is no conversion of matter into energy.

To be sure, there are processes in which matter can be created or destroyed. In the reaction between an electron and a positron, for example, both particles are annihilated and two photons are formed:

$$e^- + e^+ \rightarrow 2\gamma$$

Nevertheless, we conclude that it is not generally true that matter is converted to energy (or energy into matter) in reactions. If matter is measured by the moles of atoms or nucleons present, the quantity of matter is unchanged in all chemical reactions and in many nuclear reactions.

Paul K. Andersen *is Associate Professor of Chemical Engineering at New Mexico State University. He received his BS from Brigham Young University and his PhD from the University of California at Berkeley, both in chemical engineering. His research interests include electrochemical engineering and process simulation. He is author of* Just Enough Unix *(McGraw-Hill, 2000) and coauthor of* Essential C *(Oxford, 1995).*



INERTIAL MASS AND ENERGY

Of course, Einstein's equation refers to mass, not moles. It is worthwhile to consider exactly what is meant by *mass*. In classical mechanics, mass is a measure of two attributes of a material body:

1. The body's resistance to acceleration by external forces ("inertial mass")
2. The force the body experiences in a gravitational field ("gravitational mass")

The title of Einstein's 1905 paper[2] clearly shows that he was interested in the first concept: "Does the Inertia of a Body Depend on its Energy Content?" (Ist die Trägheit eines Körpers von seinem Energienhalt abhängig?)

The product of inertial mass and velocity is the momentum of a body. Newton's second law of motion can be written as a momentum balance, relating the inertial mass and velocity to the force exerted on the body

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} \tag{3}$$

It has been customary in classical mechanics to regard the mass of a body as a constant, independent of time or velocity (provided the body is not losing or gaining matter). Thus, the mass is usually taken out of the derivation in Eq. (3)

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

Einstein challenged the usual assumption that mass is independent of velocity. Using an argument based on the emission of radiant energy (see the Appendix), he derived a relationship between the kinetic energy and the inertial mass. He concluded, "The mass of a body is a measure of its energy content; if the energy changes by L, the mass changes in the same sense by $L/9\cdot10^{20}$, if the energy is measured in ergs and the mass in grams." In other words,

$$\Delta E = c^2 \Delta m \tag{4}$$

INTERCONVERSION OF MASS AND ENERGY?

In view of Eq. (4), would it therefore be accurate to say that *mass* and energy are interconvertible in reactions? The answer is still no. If mass and energy were interconvertible, we would expect a negative sign to appear in the equation

$$\Delta E = -c^2 \Delta m \quad (?)$$

But the mass and energy increase or decrease together, so ΔE and Δm must have the same sign.

Consider once again the fission of uranium, as described by Eq. (2). Suppose the reaction is carried out in a closed, adiabatic container, which allows no work or heat exchange with the surroundings. In that case, $\Delta E = 0$, and Eq. (4) yields $\Delta m = 0$. *The reaction occurs without any change in the mass of the system.*

What happens physically is that some of the energy stored in the uranium nucleus is converted to kinetic energy of the fission products. The temperature of the system rises; but so long as no energy is exchanged with the surroundings, the overall energy of the system does not change. Therefore, according to Eq. (4), the mass does not change.

On the other hand, suppose the thermal energy is withdrawn from the system as it is generated by the fission reaction. According to Eq. (4), this results in a decrease in the mass of the system: $\Delta E < 0$ implies $\Delta m < 0$.

Note that it is the withdrawal of energy from the system that causes the mass to decrease; there is no conversion of mass to energy in the reaction itself. Indeed, if we were to add energy to the system—such as by heating it or accelerating it—the mass would increase again.

FORMS OF ENERGY

Is mass then a form of energy? When speaking of forms of energy, we typically mean kinetic, potential, and internal energy. The total energy of a system may be taken as the sum

$$E = E_K + E_p + U$$

If mass were simply another form of energy, we would have to add another term to the equation

$$E = E_K + E_p + U + mc^2 \quad (?)$$

This is incorrect. According to Einstein, mass is a measure of the energy of the system, not a separate kind of energy. Hence, it would be proper to write

$$m = \frac{E}{c^2} = \frac{1}{c^2}\left(E_K + E_p + U\right) \tag{5}$$

Note that the mass varies with kinetic energy and therefore with velocity. We shall return to this point later.

CONSERVATION OF MASS AND ENERGY

An oft-repeated assertion is that Einstein's special theory of relativity modifies the principles of mass and energy conservation. This is only half true. Consider the general balance equation for an extensive quantity in a control volume:

(Rate of accumulation) = (net input rate) + (net generation rate)

For the energy, E, of the system, the balance equation takes the form

$$\frac{dE}{dt} = \sum_i \dot{E}_i + \dot{E}_{gen}$$

where a dot over a variable indicates a rate, and the summation is taken over the boundaries of the control volume. In thermodynamics, we recognize three ways for energy to cross the boundaries: by heat transfer, by work interactions, and by material flows. Therefore, the energy balance can be written

$$\frac{dE}{dt} = \sum_i (\dot{Q} + \dot{W} + \dot{m}\hat{E})_i + \dot{E}_{gen} \tag{6}$$

where

$\dot{Q}$ rate of heat transfer through boundary i

$\dot{W}$ rate of work through boundary i
$\dot{m}$ rate of material flow through boundary i
$\hat{E}$ energy per unit mass of material

Einstein assumed that energy is conserved—it is neither created nor destroyed. In other words, the energy generation rate is zero:

$$\dot{E}_{gen} = 0 \qquad \text{(Conservation of Energy)}$$

This is the assumption usually made in engineering analysis; therefore, our energy balance need not be modified to account for the effects of relativity.

We do, however, have to modify the usual engineering mass balance:

$$\frac{dm}{dt} = \sum_i \dot{m}_i + \dot{m}_{gen} \tag{7}$$

We normally assume conservation of mass

$$\dot{m}_{gen} = 0 \qquad \text{(Conservation of Mass)}$$

Relativity changes this. Recall that the mass of a system is a measure of its energy content. Dividing the energy balance, Eq. (6), by c^2 and assuming that energy is conserved, we obtain

$$\frac{1}{c^2}\frac{dE}{dt} = \sum_i \left(\frac{\dot{Q}}{c^2} + \frac{\dot{W}}{c^2} + \frac{\dot{m}\hat{E}}{c^2} \right)_i$$

Each term in this equation has dimensions of mass divided by time. Moreover, $\dot{m}\hat{E} / c^2 = \dot{m}\hat{m} = \dot{m}$. Thus

$$\frac{dm}{dt} = \sum_i \left(\frac{\dot{Q}}{c^2} + \frac{\dot{W}}{c^2} + \dot{m} \right)_i$$

This equation can be rearranged to produce

$$\frac{dm}{dt} = \sum_i \dot{m}_i + \sum_i \left(\frac{\dot{Q}}{c^2} + \frac{\dot{W}}{c^2} \right)_i \tag{8}$$

Comparing Eqs. (7) and (8) term by term, we conclude that

$$\dot{m}_{gen} = \sum_i \left(\frac{\dot{Q}}{c^2} + \frac{\dot{W}}{c^2} \right)_i \qquad \text{(Relativity)}$$

In other words, mass is "generated" in the system by heat transfer and work. Of course, $1/c^2 = 1 \times 10^{-17} s^2 m^{-2}$ is so small that the generation rate is usually negligible in practical problems.

RELATIVISTIC AND REST MASS

According to Eq. (5), the mass varies with velocity. To determine the velocity-dependence of mass, consider a closed, adiabatic system initially at rest. Suppose that a force, **F**, accelerates the system in such a way as to leave its potential and internal energy unchanged. The energy balance for the system reduces to

$$\frac{dE}{dt} = \dot{W}$$

Substituting $E = mc^2$ and $\dot{W} = \mathbf{F} \cdot \mathbf{v}$ into this equation, we obtain

$$\frac{d}{dt}\left(mc^2\right) = \mathbf{F} \cdot \mathbf{v} \tag{9}$$

The mass is related to the force by the momentum balance, Eq. (3),

$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} \tag{3}$$

Substituting this into Eq. (9), we obtain

$$\frac{d}{dt}\left(mc^2\right) = \mathbf{v} \cdot \frac{d(m\mathbf{v})}{dt}$$

Multiplying by 2m and rearranging yields

$$c^2 \frac{dm^2}{dt} = \frac{d(mv)^2}{dt}$$

Next we integrate, noting that $v = 0$ at $t = 0$. The result is

$$c^2\left(m^2 - m_0^2\right) = (mv)^2$$

Solving for m, we obtain at last

$$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2 / c^2}} \tag{10}$$

In this equation, m is the inertial mass, sometimes called the *relativistic mass;* and m_0 is the *rest mass,* which is the mass of the system at $v=0$.* At velocities much lower than the speed of light, $\gamma \approx 1$ and the relativistic mass coincides with the rest mass. This is usually the case for engineering problems.

CONCLUSION

We have seen that Eq. (1), Einstein's mass-energy equation, does not predict the interconversion of matter and energy in chemical or nuclear reactions. In fact, the constituents of matter are conserved in chemical reactions and in many nuclear reactions. Nor are *mass* and energy interconvertible. Instead, what Einstein showed was that the mass of a body is a measure of its energy content; consequently, the mass increases when the energy does. Because energy is conserved, there is no need to change our usual energy balance. But in some cases it may be desirable to modify the mass balance to account for the dependence of mass on energy.

We may well ask whether any of this matters, since chemical engineers rarely if ever encounter problems in which relativistic effects are significant. There are at least three reasons why it is important. First, if we are going to mention the theory in our classes or textbooks, we should try to get it right. Second, our students may in the future have to deal with problems in which a sound understanding of $E = mc^2$ is

** In recent years, the preference of many physicists has been to define the rest mass as* ***the*** *mass of the system, and to drop the subscript 0. The mass, m, then becomes independent of velocity, which may be considered an advantage; on the other hand, the factor γ must be included explicitly in many equations. For a lively discussion of this issue, see references 4 and 5. This paper has adhered to the more traditional definition, in which the mass, m, is the inertial or relativistic mass.*

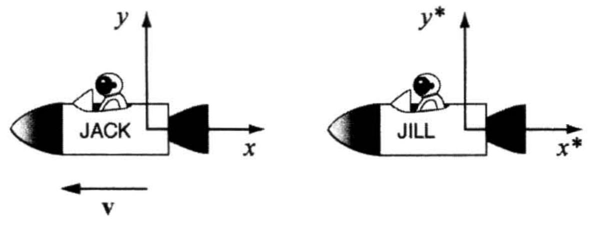
required. Finally, the Special Theory of Relativity is one of the major scientific discoveries of the 20th century. It could be argued that no scientist, engineer, or mathematician can be truly educated without a proper understanding of this theory.

REFERENCES

1. Brescia, Frank, et al., *Fundamentals of Chemistry,* Academic Press (1966)
2. Einstein, A., "Does the Intertia of a Body Depend on Its Energy Content?" (Ist die Trägheit eines Körpers von seinem Energienhalt abhängig?), *Annalen der Physik,* **17** (1905)
3. Einstein, A., "On the Electrodynamics of Moving Bodies" (Zur Elektrodynamik bewegter Körper), *Annalen der Physik,* **17** (1905)
4. Okun, L.B., "The Concept of Mass," *Physics Today,* p. 31, June (1989)
5. Rindler, W., *et al.,* "Letters," *Physics Today,* p. 13, May (1990) ❐

APPENDIX: Derivation of $\Delta E = c^2 \Delta m$

Consider two astronauts, Jack and Jill, riding their space scooters far out in interstellar space. (Space scooters had not been invented when Einstein published his derivation in 1905, but his argument was essentially the same as what follows.) Jack is moving away from Jill at a constant velocity **v**. For the purposes of our analysis, we define two coordinate systems, as shown in the Figure. The (x,y)-coordinate system is attached to Jack and moves with him; the (x*,y*)-coordinate system is attached to Jill. The systems are oriented so that the x-axis and x*-axis are parallel to the direction of **v**.



We want to calculate the energy of Jack and his scooter. To do so, we must specify which coordinate system we have in mind. Relative to Jill's (x*, y*) system, Jack is moving at speed v, giving him a kinetic energy $\frac{1}{2}mv^2$. Relative to his own (x,y) system, Jack is not moving, so he has no kinetic energy. In either system, Jack and his scooter have the same internal energy. Thus, the difference between Jill's view and Jack's view is

$$E^* - E = \tfrac{1}{2}mv^2 \qquad \text{(A1)}$$

Now suppose Jack activates his laser beacon, which fires two pulses of light. One pulse has energy L/2 and is emitted at an angle θ relative to the x-axis; the other also has energy L/2, but is emitted in the opposite direction.* Jack's velocity does not change, but the internal energy of Jack and his scooter decreases by the sum of the energies of the light pulses

$$E_2 - E_1 = \Delta E = -\left(\frac{L}{2} + \frac{L}{2}\right) = -L \qquad \text{(A2)}$$

Jill once again sees things differently. As Einstein showed in a previous paper on relativity,[3] the energies of the light pulses appear from Jill's standpoint to be

$$\frac{L}{2}\left(\frac{1+v\cos\theta}{\sqrt{1-(v/c)^2}}\right) \quad \text{and} \quad \frac{L}{2}\left(\frac{1-v\cos\theta}{\sqrt{1-(v/c)^2}}\right)$$

Therefore, Jill computes the change in Jack's energy to be

$$E_2^* - E_1^* = -\left[\frac{L}{2}\frac{1+v\cos\theta}{\sqrt{1-\left(\frac{v}{c}\right)^2}} + \frac{L}{2}\frac{1-v\cos\theta}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\right] = -L\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad \text{(A3)}$$

Subtracting Eq. (A2) from Eq. (A3), we obtain

$$E_2^* - E_1^* - (E_2 - E_1) = -L\left[\frac{1}{\sqrt{1-(v/c)^2}} - 1\right]$$

Regrouping the terms on the left-hand side of the equation yields

$$\left(E_2^* - E_2\right) - \left(E_1^* - E_1\right) = -L\left[\frac{1}{\sqrt{1-(v/c)^2}} - 1\right] \qquad \text{(A4)}$$

Referring back to Eq. (A1), we see that the left-hand side of the equation equals the change in kinetic energy of Jack and his scooter relative to Jill's (x*,y*) system. Moreover, Eq. (A2) shows that -L = ΔE. Thus, we can rewrite Eq. (A4) as

$$\Delta\left(\tfrac{1}{2}mv^2\right) = \Delta E\left[\frac{1}{\sqrt{1-(v/c)^2}} - 1\right] \qquad \text{(A5)}$$

Einstein made use of the approximation

$$\frac{1}{\sqrt{1-(v/c)^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2$$

Substituting this into Eq. (A5), we obtain

$$\Delta\left(\tfrac{1}{2}mv^2\right) = \Delta E\left[\frac{1}{2}\left(\frac{v}{c}\right)^2\right] \qquad \text{(A6)}$$

But if Jack's velocity does not change, $\Delta\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\Delta m$, and Eq. (A6) becomes

$$\Delta m = \Delta E / c^2 \qquad \text{(A7)}$$

This is the result Einstein obtained in 1905.

* *Why two laser pulses? As Einstein noted, light carries momentum. If Jack fired only one pulse, it would tend to accelerate him in the direction opposite the direction of the light. By using two equal but opposite pulses, there would be no acceleration.*