

HEAT EXCHANGER LAB FOR CHEMICAL ENGINEERING UNDERGRADUATES

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Chemical Engineering Laboratory is a required course in the undergraduate chemical engineering curriculum at The University of Akron to give students hands-on experience with the fundamental chemical engineering concepts of transport phenomena, thermodynamics, and reaction kinetics, while also designing experiments, collecting/analyzing data, and presenting their results. This course is taken in the sixth academic semester, with Mass Transfer Operations as a prerequisite and taken concurrent with Chemical Reaction Engineering and Fluid & Thermal Operations as corequisites. The heat exchanger laboratory described in this paper is one of several laboratory experiences of the 15-week course. Since students do not get extensive experience with the application of scale-up principles outside of a few discussions in the lecture courses, this heat exchanger lab was seen as an opportunity to teach the students how to apply scale-up concepts.

Proper scale-up applies the principles of geometric and dynamic similarity to obtain accurate prediction of the tank performance between the two tank sizes. Geometric similarity requires ratios of characteristic length in the tank geometry, such as impeller diameter to tank diameter or liquid depth to tank diameter, to be the same for the small- and large-scale tanks. Dynamic similarity requires the ratios of forces and ratios of energies, defined by dimensionless groups such as Reynolds number and Prandtl number, to be the same between the small and large tanks.^[1-3]

The heat exchanger laboratory had two parts (described in detail in later sections). In Part 1 the students observed and compared performances of two geometrically similar but different size stirred tanks having very simple internal coil heat exchangers. They applied energy balances to evaluate and compare the performances and to deduce the scale factor for predicting tank performance. This comparison of the operation

of two different scale tanks reinforced the concepts of scale-up and gave the students confidence in scale-up application.

In Part 2, the students designed and fabricated their own heat exchange coils and tested them in a small 10 gallon tank. The students were permitted to use baffles, extended surfaces, or

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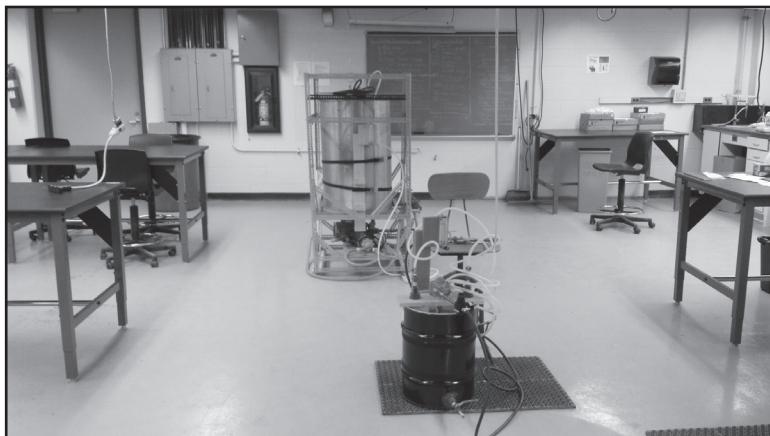


Figure 1. Laboratory setup with a 10 gallon tank (foreground) and a 50 gallon tank (background).

different tube lengths and diameters compared to the simple design used in Part 1. The students applied scale-up principles to predict performance of a larger scaled-up tank using their coil design. Due to resource limitations the students did not physically test their designs on a larger tank but they supported their design predictions through the principles of scale-up in their oral presentations.

The learning objectives of the laboratory experience included:

- Hands-on experience with cutting, bending, and connecting tubes.
- Hands-on experience in attaching and using fittings, valves, flow meters, thermocouples, and stirrers.
- Interpretation of experimental results and sources of experimental errors to link what they learned in classrooms about fluid flow and heat exchange to a practical application.
- Application of engineering concepts to an unsteady state process.
- Application of concepts of scale-up and dimensional analysis.
- Design of a workable solution to a problem with multiple constraints.

Specific laboratory activities and tasks included:

- Preparation of technical drawings.
- Use of tools to fabricate the internal heat exchange coil.
- Preparation of an experimental plan and lab report.
- Analysis of the experimental data.
- Oral presentation of the experimental results and prediction of scale-up performance of a large tank based on the student-designed heat exchanger coil.

The laboratory course was 15 weeks long. The students were divided into Monday/Wednesday groups and Tuesday/Thursday groups. The groups met for three hours on each of their assigned group days for six hours of laboratory time each week. All of the students attended a common one-hour

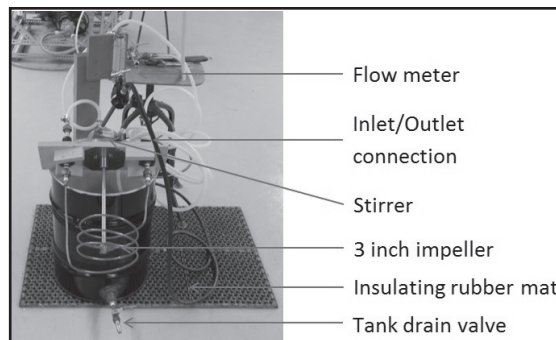


Figure 2. Ten gallon stirred tank for the students to use for their experiments. The stirrer and example coil are positioned outside of the tank for the purpose of the photograph.

lecture period on Wednesdays that covered a range of topics pertinent to the course and developmental topics for the chemical engineering degree. In the first week of the course the laboratory periods were used for safety instructions and administrative functions. In the last two weeks of the course the laboratory periods were used to complete lab activities, make up for unfinished lab activities, and complete administrative activities.

The heat exchange experiment was one of four laboratory activities conducted during weeks 2 through 13 of the course. The four laboratory activities each lasted three weeks with one-fourth of the students rotating through each activity every three weeks. In the most recent offering of the course (Spring semester 2015) the students were organized into four teams of three students each in each rotation. The three-person teams were an ideal size because there were enough students to effectively conduct the experiments and all students contributed to the team activities.

COMPARISON OF HEAT EXCHANGE PERFORMANCE OF TWO SCALED STIRRED TANKS

Heat exchange performance was compared between a 10 gallon stirred tank and a 50 gallon stirred tank. The two tanks are shown in Figure 1. A closer view of the 10 gallon tank is shown in Figure 2. Both tanks were geometrically similar and were equipped with geometrically similar simple copper tube coils through which cold water flowed. The tanks were equipped with air-pressure driven geometrically similar propellers to agitate the tank water and with rotameters to measure the cooling water flow rate through the heat exchange coils. Hand-held thermocouples were provided to measure water temperatures over time. A digital strobe light was used to measure the rotation rate of the impeller. The impeller rotation rate was controlled via an air pressure regulator valve.

The scale factor, S , between the two tanks is defined as the ratio of the tank diameters and had a value of 1.64. The 3 inch and 5 inch diameter three-blade propellers used in the small

and medium tanks, respectively, had nearly the same scale factor (1.67). Internal cooling coils were fabricated of 1/4 inch and 3/8 inch copper tubes with scaled geometries (tube length, same number of tube coil turns, scaled coil diameter, and located at the same scaled positions in the tanks). The ratio of the outside diameters of the copper tubes of 1.50 was not the same as 1.64 but close enough for the demonstration.

The demonstration started with the tanks filled with hot water (approximately 50 °C) to a depth equal to the respective tank diameters. Cooling water flowed through the coil at scaled flow rates for the two tanks to give scaled performance. The tanks cooled in about 20 and 40 minutes respectively to within a few degrees of the cooling water inlet temperature. The students plotted and evaluated the temperature–time data to determine the characteristic performance constant, c

$$c = \frac{\dot{m}}{M} \left(1 - \exp \left(\frac{-U_o A_o}{\dot{m} C_p} \right) \right) \quad (1)$$

in the performance equation (see Appendix for derivation)

$$-\ln \left(\frac{T - T_{in}}{T_o - T_{in}} \right) = ct \quad (2)$$

where T is the tank temperature at time t , T_{in} is the cooling water inlet temperature (constant), and T_o is the initial tank temperature.

Theoretically, it can be shown that if the geometric properties (tank diameter, tube diameter, and impeller diameter) all scale by S , such that $d_{large} = d_{small} * S$, then the performance constants scale by S^2 ,

$$\frac{c_{small}}{c_{large}} = S^2 \quad (3)$$

The students collected the experimental data, plotted the results, and calculated the ratio $\frac{c_{small}}{c_{large}}$. They discussed factors that contributed to error in the measurements. In situations when the ratio $\frac{c_{small}}{c_{large}}$ significantly deviated from S^2 the students further discussed how they would improve the experiments to ensure operating conditions were consistent with model constraints.

The comparison helped prepare the students for running and evaluating their own experiments by introducing them to the instruments and to the tasks they need to do during the experiments. For their own experiments the students were required to change the coil design shape and dimensions. The students were allowed to use fins, extend surfaces, and add baffles. In

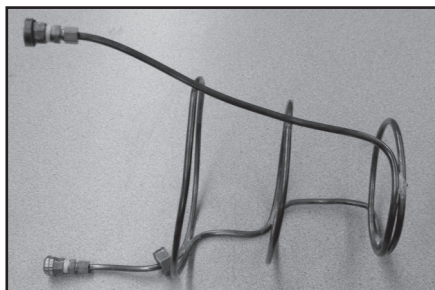


Figure 3. A typical student-designed and -fabricated 1/4 inch copper tube coil.

all cases the student teams kept the designs simple due to the limited time available for fabrication.

STUDENT-DESIGNED INTERNAL HEAT EXCHANGE COILS

The students were given the following scenario:

The heat exchanger design team is part of an engineering consulting company. A client of the consulting company wants to retrofit a stirred tank bioreactor with a heat exchanger to cool the tank contents from 70 °C to less than 25 °C. The tanks contain 25 m³ of aqueous liquid. Cooling water at 15 °C is available on-site but with a constrained maximum flow rate of 10 liters per minute for accomplishing the heat exchange.

The teams used this information to propose a design and run small-scale tests to predict the time needed to cool the contents of the bioreactor. The results of the experiments and the predicted cooling time were presented to the customer with a scaled-up design, performance prediction, and cost estimate.

For the experiments the students were provided a 10 gallon tank equipped with a 3 inch propeller stirrer for the small scale testing, as shown in Figure 2. Hot tap water at about 50 °C was used as the tank fluid, and cold tap water at about 5 °C to 15 °C was used as the cooling water. Brass union connections were used at the top of the tank for attachment of the students' heat exchangers. The teams were allowed to choose either 0.25" or 0.375" outside diameter copper tubing for fabrication of their coils. The coil design (size of coil turning radius, number of coils, shape of coil, etc.) dictated the total coil length. Based on prior experience, the students were encouraged to keep the total tube lengths less than 4 m, otherwise the cooling water temperature approached the tank water temperature and the extra tubing was ineffective in the heat exchange. With the above information, the students made a technical drawing of their heat exchanger coil design that was reviewed by the teaching assistant and a technician prior to construction of the coil. In their reviews, the teaching assistant and technician offered suggestions to improve or simplify the construction effort of the designs proposed by the students.

Once the design was approved by the teaching assistant and the technician, the students were given fittings and copper tubing. Fabrication was done in a machine shop with tools available for cutting and bending the tubing. Sometimes the bends in the coil were large and students found objects of appropriate diameter around which they could bend the tubes. A technician was available to assist if needed and to teach the students proper and safe use of the tools. Machining (lathe, mill, etc.) was available but seldom needed.

Teams were allowed to interact and observe each other during fabrication of the coils. A typical student coil is shown in Figure 3.

Experimental plan

Before running the experiments, the teams prepared experimental plans that described the procedures to follow, measurements to be recorded, and a description of how the data would be evaluated. The plan was reviewed by the teaching assistant prior to running any experiments, to ensure plans could be run safely and to check for any items the teams may have overlooked.

The students used the given parameters of the hypothetical large scale tank to scale down the operating conditions at which the small scale tanks were run. A constraint was placed on the large tank that the maximum cooling water flow rate available was 10 liters per minute. This constraint forced the students to scale the flow rate of the cooling water for the small tank based on constant Reynolds number between the scales. Since water was used as the tank fluid and the cooling fluid at both scales the fluid properties were assumed to be the same in the scale-up calculations used to determine the appropriate flow rates and impeller rotation rates for the experiments.

Since the scale factor, S , between the small and large tanks was defined as the ratio between the tank diameters, and the fluid height in the tank was equal to the diameter,

the students determined S to be $S = \left(\frac{V_{\text{large}}}{V_{\text{small}}} \right)^{0.333} = 8.91$. To

find the flow rate inside the tube, the Reynolds number for

flow was held constant ($Re_{\text{large}} = Re_{\text{small}}$) where inside of the

tube, $Re = \frac{\rho v d}{\mu} = \frac{4\rho Q}{\pi d \mu}$, hence the flow rates were related

as $Q_{\text{large}} = S * Q_{\text{small}}$. To find rotation rate, the tank agitation

Reynolds number was held constant ($Re_{o1} = Re_{o2}$) for the

stirrer, $Re_o = \frac{n d_p^2 \rho}{\mu}$, hence the stirrer rates were related by

$$n_{\text{small}} = S^2 n_{\text{large}}$$

After fabricating their coils, the students attached their coils to the 10 gallon tank, checked for leaks, filled the tank with hot water, set the appropriate rotation rate with aid of the digital strobe light, set the appropriate cooling water flow rate with a rotameter, and recorded their temperature data over time. The thermal energy balance equations from transport phenomena provide a relation to determine the overall heat transfer coefficient, which can be scaled appropriately to make a prediction of the cooling time for a larger tank, as seen in the next section.

EXPERIMENTAL RESULTS AND DISCUSSION

The students recorded tank and cooling water temperatures over time. They plotted the dimensionless temperature versus

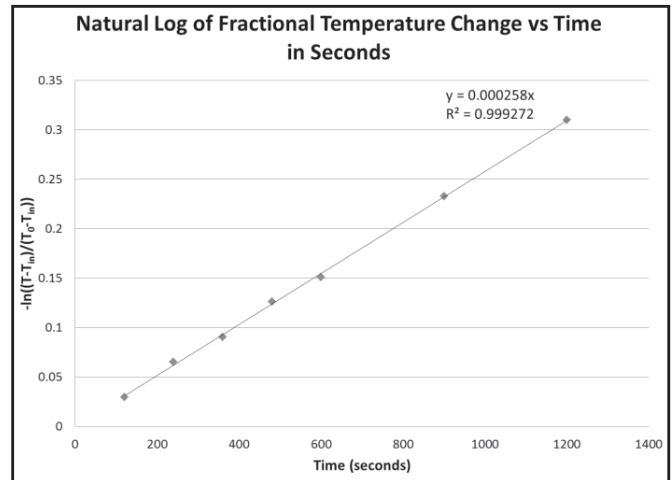


Figure 4. Example plot of the natural log of dimensionless temperature versus time using data measured from a small tank experiment. The slope of the fitted line (here 0.000258/s) was equal to the performance constant c_{small} .

time to determine the performance constant, c , as the slope of a linear fit of the data through the origin, as indicated by Eq. (2). An example plot is shown in Figure 4.

Scale-up

From c_{small} for the small tank, and the scale factor S , the constant c_{large} for the large tank was determined from Eq. (3). For the example data in Figure 4, $c_{\text{large}} = 3.25 \times 10^{-6}$. The cooling time for the large tank based on the experimental data in Figure 4 can be determined from Eq.(2) to be $t_{\text{large}} = 551,000$ s = 6.4 days.

Typical coil designs by the teams had large tank cooling times ranging from 4 to 7 days. This may seem to be a long time. During the presentations the students were asked to discuss why it seemed to take so long to cool the large tanks (due to assumptions of negligible heat loss to surroundings, limited tube length, and limit on cooling water flow rate), what factors controlled the cooling (cooling water temperature and flow rate, and surface area for heat transfer), and what recommendations they would give the client to speed up the cooling time (increase cooling water flow rate, use a longer tube coil, and consider pumping the hot tank fluid through a shell and tube exchanger to another tank).

Error analysis

Several sources of error in the experiments were identified. Variations in temperature of the cooling water from the building supply, if significant, could affect the rate of cooling of the tank water. Multiple thermocouples were used, and sometimes the thermocouples gave slightly different readings. For designs that needed a lower rotation speed of the propeller, the motor sometimes had difficulty maintaining a steady rotation speed. Human error of improper or inconsistent data collection was

observed, particularly if different students took measurements in different experimental runs. Some of the students had difficulty using the digital strobe light to measure, adjust, and control the rotation rate of the propeller.

EDUCATIONAL ASSESSMENT

The evaluation of the students' effort in this laboratory section was done by grading of the student presentations. A list of the topics covered in the presentation and guideline for assigning points for the grades are listed in Table 1. The list was provided to the students at the start of the three-week lab session so they knew what was expected of them in the final presentations.

In the final presentation the students presented the topics in the roles of the Chief Executive Officer (CEO), Chief Technical Officer (CTO), and Chief Financial Officer (CFO) or Vice President of Marketing. Each student earned up to 30 total points based on their individual performance and up to 20 total points for the team or group score. Information was given to the students on the first day of the three-week lab session that indicated the expected content for each portion of the presentation. The students had prior instruction in economic analysis from a Process Economics course from the chemical engineering department in their fifth academic semester. The students also had prior marketing analysis experience from a Project Management and Teamwork class in their first, third, and fifth academic semesters where they work in a vertically integrated team consisting of students from all levels within the program where marketing is one of the required tasks.^[4] The other lab experiences in this course required the students to complete detailed written experimental reports, short executive summaries, and technical presentations. The economic and marketing analysis in the heat exchanger lab was used to give the students a more well-rounded experience from the course as a whole.

The lab instructor and teaching assistant played the roles of a potential customer during the team presentations and asked questions relevant to the expected content as well as questions to assess the students' knowledge of the fundamentals of heat transfer.

Most students initially struggled explaining the heat exchange performance of the stirred tank because the tank operated as an unsteady state process while the cooling water flowing through the copper tube performed as a quasi-steady state process (the time rate of change of the temperature of the cooling water was very small compared to the rate of temperature change along the length of the coil). This may be because most of the theoretical coursework the students have taken to this point focused on steady state processes. Some students struggled with the fundamentals of scale-up, and had only a shallow understanding of why the dimensionless groups, such as the Reynolds Number, are held constant between the small- and large-scale tanks.

| TABLE 1 Example evaluation rubric for student presentations | |
|--|--|
| CEO | Executive summary (key points) Business model SWOT analysis (strengths, weaknesses, opportunities, threats) Why is your design competitive? Primary competitors? |
| CTO | Tech overview Performance of heat exchanger Experiments (objectives, data collection, analysis of results) Error analysis, difficulties Justification of assumptions made |
| CFO/VP Marketing | Marketing plan Economics of design and operations Profit margin |
| INDIVIDUAL | 5 pt. – Delivery/eye contact? 5 pt. – Knows material 5 pt. – Quality of slides/handouts 5 pt. – Content, accuracy, info appropriate 5 pt. – Q&A, answers questions directly and professionally 5 pt. – Info relevant to audience? |
| TEAM | 5 pt. – Presentation organization? 5 pt. – Team works as a team? 5 pt. – How well were presentation messages delivered? 5 pt. – Can team answer questions? |

Students were asked how they felt about the lab experience and if they would recommend any changes. Feedback was positive; they felt the presentation format and technical discussions helped them understand how their coursework relates to a real-world problem, and they felt the hands-on experience to fabricate their own heat exchange coils was fun and valuable. They enjoyed the ability to run a simple experiment to demonstrate and apply the scale-up concepts. Some students would prefer more room for creativity with the exchanger design and the presentation. The students did not feel overwhelmed with the material they were asked to prepare for the final presentation and felt well prepared for economic and marketing discussion based on prior coursework.

RECOMMENDATIONS

The enthusiasm of the students to run this experiment combined with the simplicity of the experimental setup demonstrates the success of this laboratory experience. A number of variations could be incorporated into the laboratory exercise if time and resources permit. Alternative experiments could be run such as to compare and evaluate externally insulated versus non-insulated tanks, effects of stirrer geometry, and effects of fouling on the cooling coils. Students could explore variations in the heat exchange coil designs to optimize (minimize) the required cooling time. Systematic experiments could be run to explore the influences of single parameters such as length of the coil.

CONCLUSIONS

In conclusion, this laboratory experience was successful in teaching the students through a hands-on example of heat transfer and scale-up while being a simple enough experiment to run in three weeks of class time. The students successfully fabricated copper tube coils for cooling hot water in a stirred tank. The students applied principles of scale-up and analyzed data from a small tank to predict performance of a scaled large tank. The cooling of the stirred tank had multiple constraints and required students to design a workable solution taking into account the scale and the unsteady state performance of the stirred tank. The experimental results and prediction of performance of the large tank were reported in a presentation format. The student teams generally performed well. Student feedback on the laboratory exercise was positive.

ACKNOWLEDGMENTS

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APPENDIX: DERIVATION OF EQ. 1

To derive Eq. (1), we start with the thermal energy balance from *Transport Phenomena*^[5]:

$$\text{Energy Balance: } \frac{dU_{\text{tot}}}{dt} = -\Delta(\dot{m}\hat{u}) + \dot{Q} + E_c + E_v \quad (\text{A1})$$

where \dot{m} is the mass flow rate of the cooling water in the coil (a constant). The last two quantities on the right side are defined as $E_c = \int_V P(\nabla \cdot \mathbf{v})dV$ that accounts for the internal energy generation due to fluid compression, and $E_v = \int_V \boldsymbol{\tau} : \nabla \mathbf{v}dV$ that is the generation due to viscous dissipation. The water is considered incompressible, constant density, hence from the mass continuity equation $\nabla \cdot \mathbf{v} = 0$ and hence $E_c = 0$. The viscous dissipation is only important for high viscosity fluids or high shear rates; neither occur in the heat exchanger, hence $E_v = 0$.

The total internal energy is defined as

$$U_{\text{tot}} = \int_V \rho \hat{u}dV \quad (\text{A2})$$

From thermodynamics

$$\hat{u} = C_v dT + \left[T \left(\frac{dP}{dT} \right)_V - P \right] d\hat{V} \quad (\text{A3})$$

Assuming the fluid is incompressible, then $d\hat{V} = 0$. Also from thermodynamics, $C_p - C_v = T \left(\frac{dV}{dT} \right)_P \left(\frac{dP}{dT} \right)_V$, for which the right side is zero (incompressible fluid), hence $C_p = C_v$. Thus, we conclude

$$d\hat{u} = C_p dT \quad (\text{A4})$$

By combining Eq. (A2) with (A3) and differentiating with respect to time the time rate of change of the total internal energy becomes

$$\frac{dU_{\text{tot}}}{dt} = \rho C_p V \frac{d\bar{T}}{dt} \quad (\text{A5})$$

where the volume averaged temperature is defined as

$$\bar{T} = \frac{1}{V} \int_V TdV.$$

Combining Eq. (A5) with (A1), and the total mass of fluid $M = \rho V$, the energy balance becomes

$$MC_p \frac{d\bar{T}}{dt} = \dot{m}\Delta(\hat{u}) + \dot{Q} \quad (\text{A6})$$

Energy balance on the coil: The convection and heat conduction terms dominate in the coil. The time rate of change of the temperature of the fluid in the coil is small in comparison, hence the accumulation term in Eq.(A6) is neglected and the energy balance becomes

$$\dot{Q}_{\text{coil}} = \dot{m}\Delta(\hat{u}) \quad (\text{A7})$$

Integration of Eq. (A4) relates the convection term to the cooling water temperatures as

$$\Delta(\hat{u}) = C_p (T_{\text{out}} - T_{\text{in}}) \quad (\text{A8})$$

Hence, combining Eq. (A7) and (A8) with (A6) gives

$$\dot{Q}_{\text{coil}} = \dot{m}C_p (T_{\text{out}} - T_{\text{in}}) \quad (\text{A9})$$

Energy balance on the tank: The energy balance on the tank is obtained in a similar way. Convection in and out of the tank is zero and the heat conduction from the tank is equal but opposite in direction to the heat transfer from the coil, $\dot{Q}_{\text{tank}} = -\dot{Q}_{\text{coil}}$, hence the energy balance reduces to

$$MC_p \frac{dT}{dt} = -\dot{Q}_{\text{coil}} \quad (\text{A10})$$

Using the design equation to define the overall heat transfer coefficient

$$\dot{Q}_{\text{coil}} = U_o A_o \Delta \bar{T}_L \quad (\text{A11})$$

where

$$\Delta \bar{T}_L = \frac{T_{\text{out}} - T_{\text{in}}}{\ln \left(\frac{T - T_{\text{in}}}{T - T_{\text{out}}} \right)}$$

is the log-mean-temperature-difference, $A_o = \pi d_o L$ is the outside tube area, and U_o is the overall heat transfer coefficient.

Combining Eq. (A9) and (A11) gives

$$\dot{m} c_p (T_{\text{out}} - T_{\text{in}}) = U_o A_o \frac{T_{\text{out}} - T_{\text{in}}}{\ln \left(\frac{T - T_{\text{in}}}{T - T_{\text{out}}} \right)} \quad (\text{A12})$$

which simplifies and rearranges to

$$T_{\text{out}} = T - (T - T_{\text{in}}) \exp \left(\frac{-U_o A_o}{\dot{m} c_p} \right) \quad (\text{A13})$$

Combining Eq. (A13) with (A9) and (A10) and some rearrangement gives

$$\frac{dT}{dt} = -\frac{\dot{m}}{M} = \left(1 - \exp \left(\frac{-U_o A_o}{\dot{m} c_p} \right) \right) (T - T_{\text{in}}) \quad (\text{A14})$$

The constant, c , is defined as the lumped parameter

$$c = \frac{\dot{m}}{M} \left(1 - \exp \left(\frac{-U_o A_o}{\dot{m} c_p} \right) \right) \quad (\text{A15})$$

Eq. (A14) becomes

$$\frac{dT}{dt} = -c(T - T_{\text{in}}) \quad (\text{A16})$$

which integrates to obtain the performance equation

$$-\ln \left(\frac{T - T_{\text{in}}}{T_o - T_{\text{in}}} \right) = ct. \quad (\text{A17})$$

□