ChE laboratory

DESIGN OF PROCESS CONTROL SYSTEMS

Using Frequency Response And Analog Simulation Techniques

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THE TIME REQUIRED to design control systems by analog simulation can be significantly reduced if initial approximations are made using frequency response methods. The tendency for most engineers is to use only one of the above techniques, the one with which they are the most familiar.

The object of this study is to present a method which will reduce the time involved in designing a control scheme for a process control system. The method itself combines the use of frequency response methods (Bode diagrams) and time domain methods (analog computer simulation) to give the most efficient design technique.

DEVELOPMENT

TO ILLUSTRATE THE METHOD, let us consider a chemical process system described by the linear model shown in Fig. 1. The system to be studied is a spray dryer used to process an emulsion to obtain a powder. Control may be effected by regulating the feed to the diffuser vanes. The air heater consists of two non-interacting transfer lags of time constant 100 seconds each. The drum behaves as three transfer lags of time constant 12.5 seconds each, and one distance velocity lag of 2 seconds. A distance-velocity lag of 3 seconds exists between changes in air temperature at the heater and its appearance at the diffuser vanes. The detecting element has a transfer lag of time constant 1 second. We will neglect the time constant of the control valve. The block diagram of the system is given in Fig. 1.

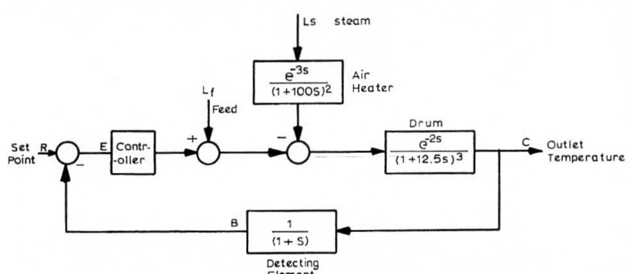


Fig. 1. Linear Model of Chemical Process System.

Note that any controller designed should reduce the steady-state error to a step input at R to an arbitrarily small value, and at the same time minimize the effect of the load of less than 10% and have reasonable stability.

For a type zero system excited by a step input the steady-state error is defined as follows

$$e_{ss} = \frac{A}{1 + K_p} \qquad (1)$$

where A = magnitude of step, K_p = position error constant defined as lim G(s) as s ⟶ o. This steady-state error is equal to R-B. What we are actually interested in is R-C. However, since there is unity gain in the feed-back path B = C in the steady-state. For our system $K_p = K_c$ which is the controller gain. Thus the steady-state specification will be met by values of K_c greater than 9. Let us now choose $K_c = 10$.

The Bode diagram of the system including controller gain, using straight line approximations, is given in Fig. 2. Using proportional control only we would have a slope of –3 at cross-

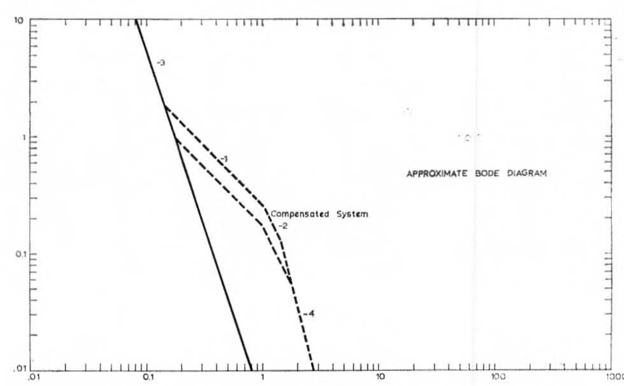
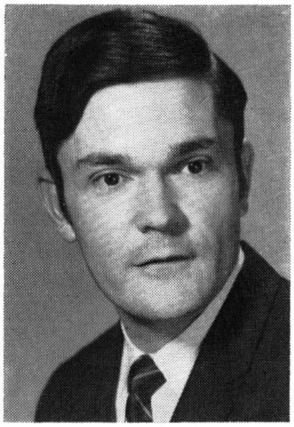


Fig. 2. Bode Diagram of System by straight line approximation.



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over which indicates the system would be unstable. However, the presence of dead-time terms requires that the phase angles be computed independently rather than from the magnitude curves using a tangent scale.

The crossover frequency occurs at 0.175 rad/sec. This is only an approximation however since we are using straight line approximations for the Bode diagram. The simulation on the analog computer will give us the true results for the model but the information from the Bode diagram is valuable in that it gives us approximate values and enables us to go to the computer where the optimal solution can be obtained by minor adjustment of the parameters.

We will now check the stability of the system using proportional control only, again by tabulating the phase angle contributions at the crossover frequency of all time constants and dead time terms.

$3 \tan^{-1}(.175)(12.5)$	$= 3 \tan^{-1}(2.185)$	$= -196.2°$
$\tan^{-1}(.175)(1)$	$= \tan^{-1}(.175)$	$= -9.9°$
	$2(.175)180/3.14$	$= -20.0°$
		$-226.1°$

The phase margin is $-46.1°$ and the system is clearly unstable.

In order to stabilize the system we must bring the slope of the curve at crossover up to a value of -1. This cannot be accomplished by a single lag-lead network (three mode controller) since the slope must be decreased by two. We will try a double lead network (two-proportional plus derivative controllers in series). The general form of the transfer function for this control system is given as

$$\frac{K_c(1 + \tau_d s)^2}{\left(1 + \frac{\tau_d s}{\gamma}\right)^2} \qquad (2)$$

where $\gamma = 10$, τ_d = controller constant, and K_c = gain.

To check for stability we must select a value of τ_d and compute the phase margin. Once again we stress that this will only be an approximation. For a first try, let $\tau_d = 7.1$. This is shown in Fig. 2. This gives a crossover frequency of 0.27 rad/sec from which we can compute the phase margin.

Tabulating the phase angle contributions we obtain

$3 \tan^{-1}(.27)(12.5)$	$= 3 \tan^{-1}(3.38)$	$= -220.5°$
$\tan^{-1}(.27)(1)$	$= \tan^{-1}(.27)$	$= -15.1°$
$2 \tan^{-1}(.27)(.71)$	$= 2 \tan^{-1}(.193)$	$= -21.8°$
$2 \tan^{-1}(.27)(7.1)$	$= 2 \tan^{-1}(1.93)$	$= +125.2°$
	$2(.27)180/3.14$	$= -30.9°$
		$-163.1°$

This gives a phase margin of 16.9°. We will make a second try in an attempt to increase the phase margin. We will let $\tau_d = 5.7$ which is also shown in Fig. 2. These values of τ_d are selected by choosing a breakpoint on the curve.

Once again tabulating the phase angle contributions we obtain

$3 \tan^{-1}(.175)(12.5)$	$= 3 \tan^{-1}(2.185)$	$= -196.2°$
$\tan^{-1}(.175)(1)$	$= \tan^{-1}(.175)$	$= -9.9°$
$2 \tan^{-1}(.175)(.57)$	$= 2 \tan^{-1}(.10)$	$= -11.4°$
$2 \tan^{-1}(.175)(5.7)$	$= 2 \tan^{-1}(1.0)$	$= +90.0°$
	$(2)(.175)\ 180/3.14$	$= -20.0°$
		$-147.5°$

This gives a phase margin of 32.5° which is a safer margin of stability. A look at the calcu-

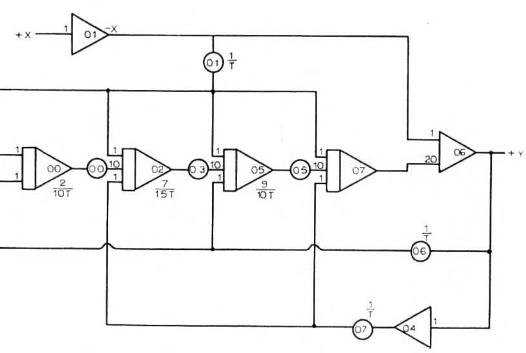

Fig. 3. Dead Time for Drum.

$P_{00} = .1000$, $P_{01} = .5000$, $P_{03} = .2322$,
$P_{05} = .4500$, $P_{07} = .5000$, $P_{07} = .5000$.

These integrators are run at times ten normal speed.

lations shows that moving the breakpoint in either direction will decrease the phase margin. If we make the breakpoint smaller we have a higher crossover frequency which yields a smaller phase margin, and if we make it larger we are back to the situation of a crossover slope of -3. Thus we are at the point of maximum relative stability. Therefore the transfer function for the compensated system is given as

$$\frac{10(1 + 5.7s)^2 e^{-2s}}{(1 + .57s)^2 (1 + 12.5s)^3 (1 + s)} \qquad (3)$$

THE NEXT STEP WILL BE to simulate the system on the analog computer. For this purpose the EAI-680 analog computer was utilized. The first phase of the study was to check the frequency

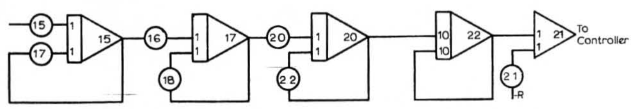

Fig. 4. Drum.

$P_{15} = 0.8000$, $P_{16} = 0.8000$, $P_{17} = 0.8000$, $P_{18} = 0.8000$, $P_{20} = 0.8000$. $P_{21} = 0.1000$, $P_{22} = 0.8000$

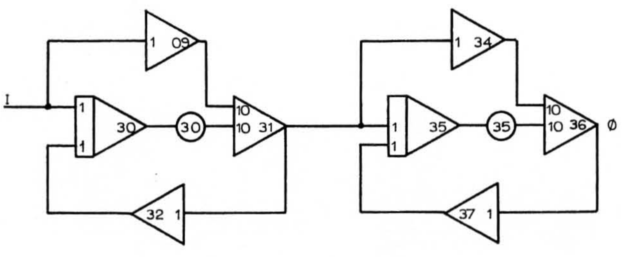

Fig. 5. Double Lead Controller.

$P_{30} = P_{35} = 0.1750$.

These integrators are run at ten times normal speed.

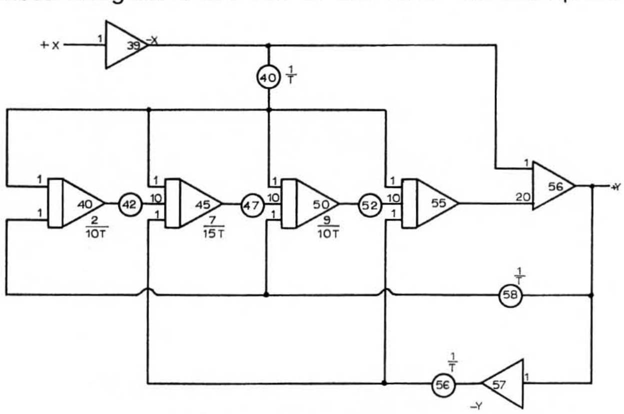

Fig. 6. Dead Time for Air Heater.

$P_{40} = 0.3333$, $P_{42} = 0.0667$, $P_{47} = 0.1558$, $P_{52} = 0.3000$, $P_{56} = 0.3333$, $P_{58} = 0.3333$.

These integrators are run at ten times normal speed.

. . . although simulation is the only method which yields exact solutions for models . . ., the use of . . . the straight line Bode diagram gives ball park estimates . . .

response by developing a Bode diagram. The next phase was to study the dynamic operating response to changes in set point and load variables, and attempt to optimize the design by varying K_c and γ. The dead time was simulated by a modified fourth order Pade delay circuit. The complete set of analog diagrams is given in the Figs. 3, 4, 5, 6, 7, and 8.

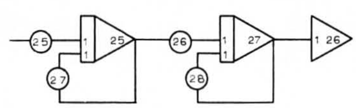

Fig. 7. Air Heater.

$P_{25} = 0.1000$, $P_{26} = 0.1000$, $P_{27} = 0.1000$, $P_{28} = 0.1000$.

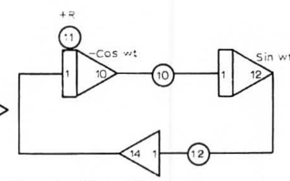

Fig. 8. Sine Wave Generator

$P_{10} = 0.1750$, $P_{11} = 0.1000$, $P_{12} = 0.1750$.

These integrators are run at ten times normal speed.

RESULTS

FROM APPROPRIATE CURVES, the open-loop frequency response of the system was determined and compared to the Bode diagram in Fig. 2.

The true frequency response of the model of the system was obtained by generating a sine wave over a range of frequencies and imposing it as an input to the set point of the system. Both the input and output of the system were recorded on a strip chart recorder. From the input and output curves the magnitude ratio and phase angle can be measured as functions of frequency. The results are plotted in Fig. 9. A comparison with Fig. 2 shows the error from the straight

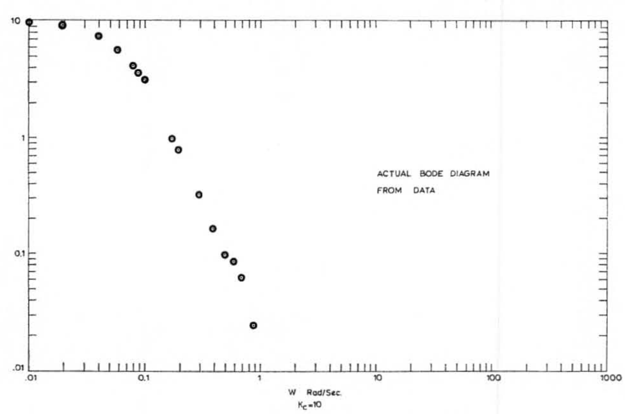

Fig. 9. Actual Bode Diagram of system.

line approximations. However, from Fig 9 we see that the crossover is about the same frequency as in Fig. 2. The phase shift at this frequency can be computed from the strip chart. In this case the phase shift was -170° giving a phase margin of +10°. This indicates that the response of the system will be stable but highly oscillatory.

The next step was to test the steady state error due to a step disturbance at the set point. A step of one volt was applied and the steady state error was calculated as 0.098 volt which is within the specification of 10% The strip chart recording of the test is shown in Fig. 10.

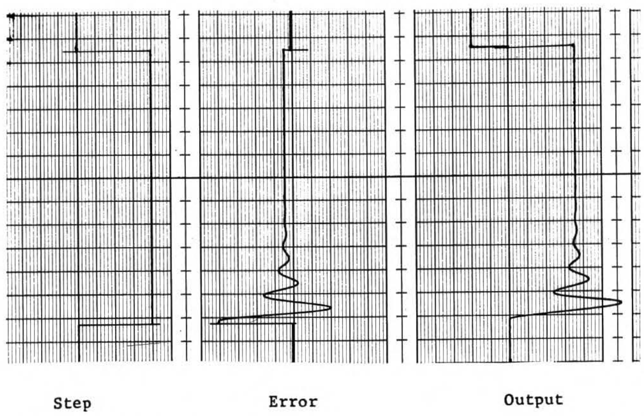

Fig. 10. Transient response of system to a step change in set point. 0.05 volts/div, 2mm/sec, $K_c = 10$.

A step disturbance was also applied at the load variable. This produced the same steady state error. This was expected since the steady state value B(t) is the same as C(t).

The overshoot can be measured from Fig. 10 and is about 60% which is somewhat high. A run was made with $K_c = 9$ which reduced the overshoot to 55% but gave a steady state error of 10% which is just on the limit of the specification. A third run was made with $K_c = 8$ which gave an overshoot of 50% and a steady state error of 12.5%. It was felt then that the original value of $K_c = 10$ was the best compromise.

Several more runs were made using this value of K_c and varying τ_d in both directions. In all cases the overshoot was worse as predicted from the original Bode diagram.

A last set of runs was made in an attempt to optimize γ. For the first run K_c and τ_d were kept the same and $\gamma = 15$ was used. This gave an overshoot of 60% which was the same as $\gamma = 10$. A value of $\gamma = 5$ was used and this gave an overshoot of 80%. Therefore the original value of γ was retained.

CONCLUSIONS

It Has Been Proven by this experiment that although simulation is the only method which yields exact solutions for models used for control system design, the use of approximations such as the straight line Bode diagram gives ball park estimates for the controller constants which are valuable starting points for the simulation study. Otherwise, one must hunt at random on the computer until the optimum values are found, which can be time consuming and wasteful. In this particular problem the values found by the approximate technique turned out to be the optimum values from the computer study. □

APL: (Continued from page 39)

education circles, it is also possible to share the resulting programs with other educators having similar interests. □

REFERENCES

1. Himmelblau, D.M., "A Course on the Optimization of Large Scale Systems." Chem. Eng. Education, 5, No. 4, pp 196 (1971).
2. Beamer, J.H., "Statistical Analysis and Simulation," Chem. Eng. Education, 5, No. 4, pp 192 (1971).
3. Franks, R.G.E., "Mathematical Modeling in Chemical Engineering," John Wiley & Sons, Inc., N.Y. (1967).
4. Kabel, R.L., "Mathematical Modeling," Chem. Eng. Education, 5, No. 4, pp 184 (1971).
5. Carnahan, B., Seider, W.D. and Katz, D.L., "Computers in Engineering Design Education, Vol. II, Chemical Engineering," The University of Michigan, College of Engineering (1966).
6. Westerberg, A.W., "Computer Aided Process Design," Chem. Eng. Education, 5, No. 4, pp 180 (1971).
7. Iverson, K.E., "A Programming Language," John Wiley, New York (1962).
8. Katzan, Harry Jr., "APL Programming and Computer Techniques," Van Nostrand (1970).
9. Falkoff, A.D. and Iverson, K.E., "APL/360: User's Manual," I.B.M. (1968).
10. Hatcher, W.S., and Rethier, P.E., "Une application du language APL au problème de démonstration de théorèmes par ordinateur," in "Colloque APL," Institut de Recherche d'Informatique et d'Automatique, Paris, France (1971).
11. de Vahl, Davis G., and Holmes, W.N., "The use of APL in Engineering Education," in "Colloque APL," Institut de Recherche d' Informatique et d'Automatique, Paris, France (1971).
12. Smillie, K.W., "Statpack 2: An APL Statistical Package," Department of Computing Science, The University of Alberta, Edmonton, Alberta, Canada.