EXPANSION AND CONTRACTION LOSSES IN FLUID FLOW

JOSEPH J. MARTIN
University of Michigan
Ann Arbor, Michigan 48104

Joseph J. Martin was educated at Iowa State, Rochester, and Carnegie-Mellon University Sc.D. '47). He is professor of Chemical Engineering and associate director of the Institute of Science and Technology at the University of Michigan. Presently he is president of Engineers Joint Council.

IT IS GENERALLY AGREED that engineering students need experience in analysis of practical problems. A situation that may be of interest to engineering educators arose when the supposedly simple problem of calculating expansion and contraction losses was encountered in a study of coolant flow through the core of a nuclear reactor. It was thought at first that one merely had to go to standard texts or handbooks to obtain the necessary information, but that proved to be not completely satisfactory, as is to be shown.

In long pipelines, skin friction losses usually predominate so that estimates of minor losses due to expansions or contractions in diameter need not be very precise. In heat exchangers with large headers and relatively small tubes and in some nuclear reactor core configurations, however, it is the expansion and contraction losses which are great and must be determined fairly accurately. In most treatments of the subject expansion losses are considered first. Referring to Fig. 1A, the mechanical energy balance is written between points 1 and 2 as

$$\frac{P_1}{\rho} + \frac{u_1^2}{2} = \frac{P_2}{\rho} + \frac{u_2^2}{2} + F_e \qquad (1)$$

where P is pressure, ρ is densiy, u is the average velocity, and F_e is the expansion loss (friction).

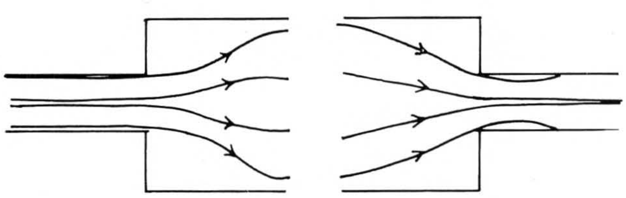

FIGURE 1.

Next the momentum balance is applied between the same points with the pressure just inside the expanded section being the same as the pressure inside the smaller section so that

$$P_1A_1 + P_1(A_2-A_1) - P_2A_2 = u_1A_1 \quad (u_2-u_1) \qquad (2)$$

The pressure difference, P_1—P_2, may be eliminated between Eqns. (1) and (2) to give the familiar Borda-Carnot relation,

$$F_e = \frac{(u_1-u_2)^2}{2} = \frac{u_1^2}{2}\,(1-A_1/A_2)^2 \qquad (3)$$

where the continuity equation $u_1A_1 = u_2A_2$, has been used. This relation was studied experimentally by Schutt [8] and later by Kays [2] and shown to be excellent for turbulent flow at Reynolds Numbers of the order of 10,000 or more in the smaller section so that velocities are fairly uniform across the upstream and downstream cross-sections. For laminar flow the losses are somewhat less, but that was not of interest in the reactor problem at hand.

Contraction losses posed a different problem, for not one book showed a method of applying overall momentum and energy balances to predict these, probably because this case is not so simple as with expansion. In the usual treatment of the subject contraction losses are calculated by the expression,

$$F_e = \frac{K_c u_2^2}{2} \qquad (4)$$

where reference is to Fig. 1B and K_c is a factor given by an empirical graph or equation. Values of K_c differ by as much as 50% between various authors and this was not acceptable for the nuclear reactor calculations. Figure 2 presents several suggested curves and many others were examined which gave the same values as one of those shown. Others, such as Bennett and Myers [1], give contraction and expansion losses in terms of equivalent length of straight pipe which is much too approximate for the desired results here.

The curve of Streeter [9] is based on century-old data, for he tabulated a contraction coefficient (area of vena contracta at 3 divided by the area at 2 or 4) which was measured by Weisbach [11]

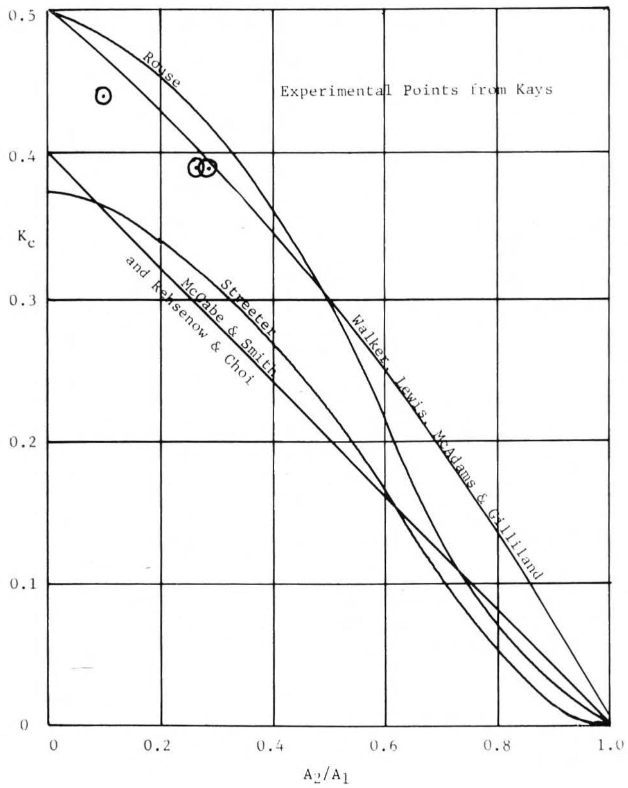

FIGURE 2.
Contraction Coefficient As Function Of Areas

The supposedly simple problem of calculating expansion and contraction losses was encountered in a study of coolant flow through the core of a nuclear reactor.

over a century ago, and inserted it into $K_c = (1/C_c - 1)^2$. However, Streeter also says, "The loss at the entrance to a pipeline from a reservoir is usually taken as 0.5 $u_2^2/2$, if the opening is square-edged." This is obviously somewhat different from a value of 0.376 $u_2^2/2$ based on Weisbach's data at $A_2/A_1 \approx 0$. Walker, Lewis, McAdams and Gilliland [10] referred to some early limited experimental data that could be represented by $K_c = 1.5(1 - A_2/A_1)/(3 - A_2/A_1)$ and it will be seen that this is good for mid-range values of A_2/A_1, but poor at higher and lower values. McCabe and Smith [5] and Rohsenow and Choi [6] took an equation, $K_c = 0.4(1 - A_2/A_1)$, which Kays [2] developed for infinite Reynolds Number and this is much less than the preceding formulation. Rouse [7] utilized two-dimensional irrotational flow analysis to obtain his results. The points on Fig. 2 are the carefully measured values of Kays [2] for both single and multiple tube arrangements for Reynolds Numbers of the order of 20,000. It is seen that none of the correlations fits the data with high precision over all ranges of the area ratio.

In view of the similarity in flows through a sharp-edged orifice and through a sudden contraction, it seemed reasonable to apply known orifice behavior to the contraction problem. An experimental study [4] of the pressures on the upstream face of a sharp-edged orifice plate showed that the average pressure on the plate differs slightly from the upstream pressure, so that the net pressure acting on the fluid between the upstream and the vena contracta can be given as $mA_2(P_1 - P_3)$. The factor m is unity for the ideal case of a perfectly uniform pressure of P_1 over the upstream orifice plate face, but differs a little from unity in actual cases. Applying the mechanical energy and momentum balances between the upstream and the vena contracta with the assumption of no losses [4] before the vena contracta gives by using $u_1A_1 = u_2A_2 = u_3A_3$, where A_3 is the cross-sectional area of the vena contracta,

$$u_2 = \frac{m}{2\sqrt{1-mA_2/A_1}}\sqrt{\frac{2(P_1-P_3)}{\rho}} \qquad (5)$$

The usual orifice equation with the approach velocity correction is written

$$u_2 = C\sqrt{\frac{2(P_1-P_3)}{\rho[1-(A_2/A_1)^2]}} \qquad (6)$$

Comparison of (5) and (6) yields

$$C = \frac{m}{2}\sqrt{\frac{1-(A_2/A_1)^2}{1-mA_2/A_1}} \qquad (7)$$

Lamb [3] reported that Kirchoff and Rayleigh had studied ideal flow in two dimensions through an orifice in a large chamber by complex variable transformation and found $C = \pi/(\pi + 2) = .6110$. The ASME Fluid Meters Report [12] showed that for a wide range of D_2/D_1 and Reynolds Numbers in turbulent flow in actual orifices C varied from 0.590 to 0.615. Thus, a round value of 0.6 may be taken for either an orifice or a sudden contraction so that Eqn. (7) becomes

$$\frac{1-mA_2/A_1}{1-(A_2/A_1)^2} = \left(\frac{m}{1.2}\right)^2 \qquad (8)$$

and m is seen to be a function only of the area ratio. In terms of m betwen points 1 and 3 in Fig. 1B the momentum balance is

$$u_1\rho A_1 (u_3 - u_1) = mA_2 (P_1-P_3) \qquad (9)$$

while the mechanical energy balance for frictionless flow as assumed in Reference [4], is

$$\frac{P_1}{\rho} + \frac{u_1^2}{2} = \frac{P_3}{\rho} + \frac{u_3^2}{2} \qquad (10)$$

If now P_1—P_3 is eliminated between (9) and (10) instead of u_3, the vena contracta velocity, as was done to get Eqn. (5), the result is

$$u_3 = u_2 (2/m-A_2/A_1) \qquad (11)$$

Between points 3 and 4 the momentum balance is

$$u_1A_1 \rho (u_4-u_3) = (P_3-P_4) A_2 \qquad (12)$$

and the mechanical energy balance is

$$\frac{P_3}{\rho} + \frac{u_3^2}{2} = \frac{P_4}{\rho} + \frac{u_4^2}{2} + F_c \qquad (13)$$

Eliminating P_3—P_4 between (12) and (13), noting that $u_2 = u_4$ and using (11) gives

$$F_c = (2/m-A_2/A_1-1)^2 \frac{u_2^2}{2} \qquad (14)$$

Comparison of (14) and (4) shows that

$$K_c = (2/m-A_2/A_1-1)^2 \qquad (15)$$

where m is the function of A_2/A_1 in (8). It is seen that fixing A_2/A_1 determines K_c. This has been done and the results compared with Kay's experimental data in Fig. 3. The agreement is within the error of the measurements themselves.

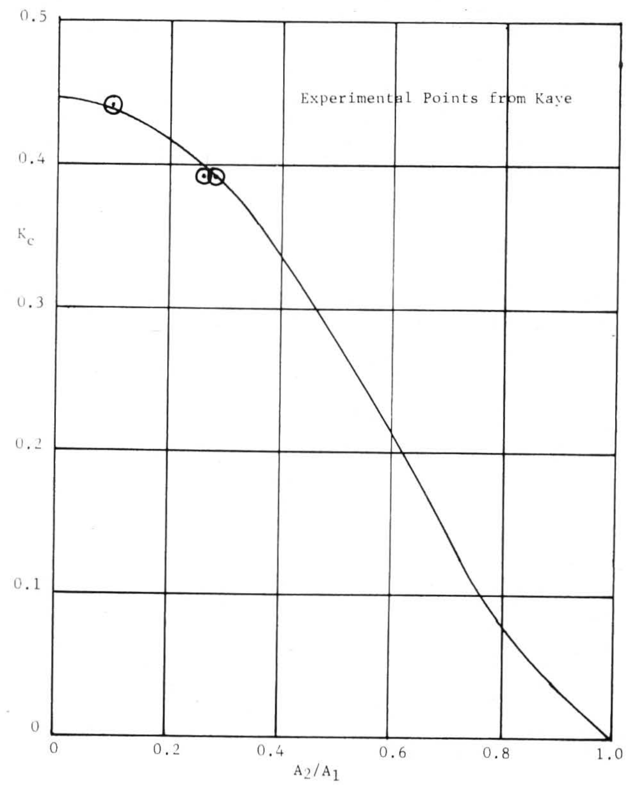

FIGURE 3.
Contraction Coefficient From Equations (8) and (15)

One concludes, therefore, that a sudden contraction is similar to a sharp-edged orifice up to the vena contracta and that the analysis by the overall mechanical energy and momentum balances along with known orifice behavior is the correct approach to this problem. Losses due to sudden contraction may, thus, be calculated with excellent precision by use of Eqns. (4), (8), and (15) or the equivalent Fig. 3.

The foregoing development assumed constant density, as for a liquid. The application can also be made to a gas if the pressure drop is not too great so that an average density may be used, since this was shown to be true in the orifice study [4]. □

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tice in independent thinking and their added self-confidence have been helpful to them in industry.

Table II. REPORT SUBJECTS

First Semester

- **Analogous commercial processes—Esterification mechanisms**
- **Possible rate equations—Proposed kinetics experimental design**
- **Interim kinetics data**
- **Final kinetics data and model evaluation**

Second Semester

- **Batch, tubular, and CSTR sizing**
- **Reactor costing**
- **Estimated boiling points**
- **Estimated vapor pressure curves**
- **Estimated vapor-liquid or liquid-liquid equilibria**
- **Final report—Plant design, product cost, and location**

The more perceptive students quickly recognize the link between theory and experiments, and they use theory successfully to design experiments. The lesson is later solidified, since experimental data must be extrapolated with theory to obtain proper design.

Eventually the usefulness of proper experimental design becomes apparent to nearly everyone. Unfortunately for many of the students, this awareness often does not come early or easily. If we were primarily interested in good results during this laboratory, we would take a more direct hand in helping students set up their experiments. Since, however, we view the laboratory mainly as a learning tool, we allow quite a bit of slipping and sliding before stepping in.

One of the unexpected results was that three-man groups were over any extended period more unstable than groups of either two or four members. Usually one or occasionally two of the three did not carry a fair share of the load. This of course led to hard feelings, even though we pointed out to the aggrieved parties that they probably learned more that way. The problem was solved by going to two-man groups. Only a very exceptional person has the temerity to leave the whole load on his partner. Conversely, the dominant partner, if one emerges, realizes that civility enlists more cooperation than alternate modes of behavior.

POSTSCRIPT

TO SOME EXTENT this approach has some flavor of reinventing the wheel, since we learned long after undertaking it that the late Professor Vilbrant at VPI had trod a similar path a number of years ago. However, the development of this sequence has educated us during the past five years perhaps more than any of our students. Our appreciation goes to those students who have volunteered advice, some of it perhaps not so well received at the time, that indicated which of our many changes were successful and which were not. No course of this type can ever stand still. We, therefore, hope that our students in the future will be equally free with their help. □

EXPANSION AND CONTRACTION LOSSES: Martin

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