THERMAL CONDUCTIVITY OF A HOTDOG

MICHAEL J. MATTESON AND
JUDE T. SOMMERFELD
Georgia Institute of Technology
Atlanta, GA 30332

PROBLEM STATEMENT

In a proposed new chemical engineering laboratory experiment, students are to determine the thermal conductivity of a hotdog. The procedure consists of inserting a thermistor into the center axis of the hotdog, about midway between its ends, and then totally immersing the hotdog in an agitated tub of boiling water. The thermistor is then used to measure the temperature increase of the hotdog with time. A sketch of this experiment is shown in Fig. 1.

The results of one such experiment are shown in Fig. 2. In this experiment the initial hotdog temperature was 70°F, and the boiling water temperature was 212°F. The density of the hotdog (diameter = 1 inch) may be taken as 50 lbs/ft^3, and its heat capacity as 1.0 BTU/lb·°F. From these data and those of Fig. 2, determine the thermal conductivity of this hotdog. It may be assumed that the surface temperature of the hotdog is the same as the boiling water temperature (that is, the convective heat transfer coefficient at this surface is very large).

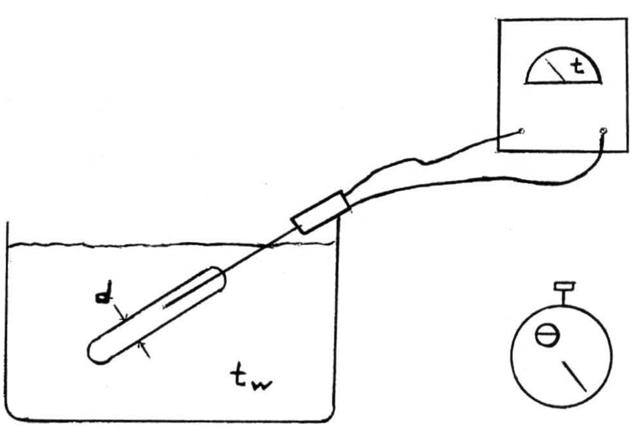


FIGURE 1. Sketch of experimental apparatus.

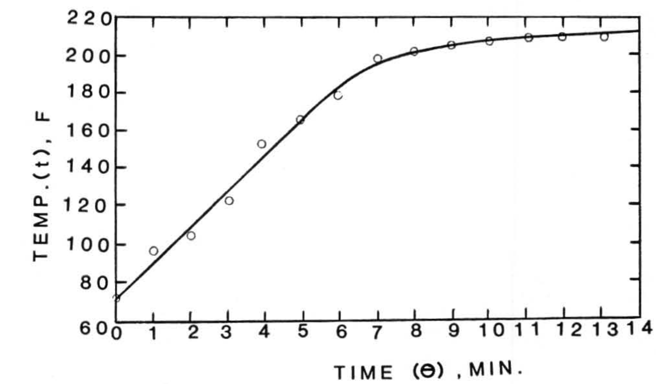


FIGURE 2. Center-line temperature (t) as a function of time (θ).

SOLUTION

The partial differential equation for unsteady-state heating (or cooling) by conduction in one direction (radial) with cylindrical geometry is well known [1]

$$\frac{k}{\rho c_p}\left[\left(\frac{\partial^2 t}{\partial r^2}\right) + \frac{1}{r}\left(\frac{\partial t}{\partial r}\right)\right] = \frac{\partial t}{\partial \theta} \quad (1)$$

If one makes the following conventional definitions

$$T = \frac{t_w - t}{t_w - t_o} \quad (2)$$

$$x = \frac{r}{r_o} \quad (3)$$

$$\alpha = \frac{k}{\rho c_p} \quad (4)$$

$$z = \frac{\alpha\theta}{r_o^2} \quad (5)$$

Eq. (1) becomes

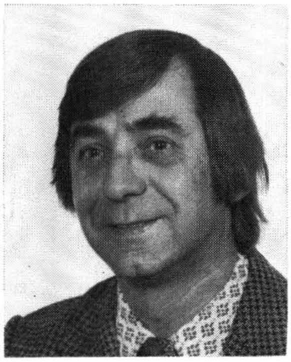
$$\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial x^2} + \frac{1}{x} \cdot \frac{\partial T}{\partial x} \quad (6)$$

The boundary conditions for this problem, in terms of the new variables, are

(i) $T(1,z) = 0 \text{ for } z > 0$ (7)

(ii) $\frac{\partial T}{\partial x}(0,z) = 0 \text{ for } z > 0$ (8)

(iii) $T(x,0) = 1 \text{ for } 0 \leq x < 1$ (9)



Michael J. Matteson is Director of the Fine Particle Technology Laboratory and professor of ChE at Georgia Tech. He received his BS and MS degrees from the University of Washington in 1960 and his D.Eng. degree from the Technical University Clausthal (Germany) in 1967. He came to Georgia Tech in 1969 after a post-doctorate at the University of Rochester. Dr. Matteson was a Fulbright Lecturer at the University of Vienna, 1974-75. **(L)**

Jude T. Sommerfeld has been a professor of ChE at Georgia Tech since 1970. He teaches courses on process control, distillation, reactor design and process design, and his research interests include energy conservation. He has also served as a consultant to numerous industrial organizations. Dr. Sommerfeld received his B.Che degree from the University of Detroit, and his MSE and PhD degrees in chemical engineering from the University of Michigan. **(R)**

Note that end effects were neglected in the derivation of Eq. (1).

Eq. (6) with its associated boundary conditions is a standard Sturm-Louiville system [2] and its solution is given by

$$T(x,z) = \sum_{n=1}^{\infty} \frac{2\exp(-\lambda_n^2 z)}{\lambda_n J_1(\lambda_n)} J_o(\lambda_n x) \qquad (10)$$

where λ_n is the n^{th} root resulting from solution of the following equation

$$J_o(\lambda) = 0 \qquad (11)$$

From Eq. (10), the expression for the center-line temperature (at $x = 0$) is

$$T(0,z) = \sum_{n=1}^{\infty} \frac{2\exp(-\lambda_n^2 z)}{\lambda_n J_1(\lambda_n)} \qquad (12)$$

In many practical calculations, it is necessary to consider only the first term in the infinite series summation of Eq. (12).

Choosing a value of $\theta = 5$ min ($= 1/12$ hr), we read from Fig. 2 a value for the center-line temperature of 166°F. Hence

$$T(0,z) = \frac{212-166}{212-70} = 0.324$$

As a first approximation, we consider only the first term on the right-hand side of Eq. (12). The first root of Eq. (11) is $\lambda_1 = 2.405$ and $J_1(\lambda_1) = 0.5191$ [3]. Equating this first term to $T(0,z)$ and solving for z, we find that $z = 0.276$. Hence

$$\alpha = \frac{r_o^2 z}{\theta} = \frac{(1/24)^2\,(0.276)}{(1/12)} = 0.00576\ \text{ft}^2/\text{hr}$$

and
$$k = \alpha\rho c_p = (0.00576)\,(50)\,(1.0) = 0.288\ \text{BTU/hr}\cdot\text{ft}\cdot{}^\circ\text{F}$$

Let us evaluate the second term in the summation of Eq. (12) to determine its significance. Here, $\lambda_2 = 5.520$ and $J_1(\lambda_2) = -0.3403$ [3]. Using the value of z determined above, we find the value of this second term to be equal to -0.000234, which is less than 0.1% of the first term.

Graphical solutions to Eq. (10) or (12) also exist in the form of the Gurney-Lurie charts. Here, $m = k/r_o h = 0$ because h is infinite, in accordance with the earlier assumption regarding the surface temperature. Again using the same center-line data point at $\theta = 5$ min, we find from the Gurney-Lurie chart for long cylinders [4] that $z = 0.28$. Hence, $\alpha = 0.058$ ft²/hr and $k = 0.29$ BTU/hr·ft·°F. □

REFERENCES

1. Carslaw, H. S. and J. C. Jaeger, *Conduction of Heat in Solids*, 2nd Edition, Clarendon Press, Oxford, England (1959).
2. Churchill, R. V., *Fourier Series and Boundary Value Problems*, McGraw-Hill Book Company, New York (1941).
3. Jahnke, E. and F. Emde, *Tables of Functions*, Dover Publications, New York (1945).
4. McAdams, W. H., *Heat Transmission*, 3rd Edition, McGraw-Hill Book Company, New York (1954).

NOMENCLATURE

- c_p heat capacity of cylinder, BTU/lb·°F
- h surface heat transfer coefficient, BTU/hr·ft²·°F
- J_o Bessel function of the first kind and of order zero
- J_1 Bessel function of the first kind and of order one
- k thermal conductivity of cylinder, BTU/hr·ft·°F
- m $k/r_o h$
- r variable radius, ft
- r_o radius of cylinder, ft
- T dimensionless temperature $= (t_w - t)/(t_w - t_o)$
- t variable temperature of cylinder, °F
- t_o initial temperature of cylinder, °F
- t_w water temperature, °F
- x dimensionless radius $= r/r_o$
- z dimensionless time $= \alpha\theta/r_o^2$
- α thermal diffusivity of cylinder, ft²/hr $= k/\rho c_p$
- λ_n n-th root of the equation $J_o(\lambda) = 0$
- ρ density of cylinder, lbs/ft³
- θ time, hrs