

LINEAR OPERATOR METHODS IN CHEMICAL ENGINEERING

By D. Ramkrishna and N. Amundson
Prentice-Hall, Inc., Englewood Cliffs, NJ 07632

Reviewed by
Sangtae Kim
University of Wisconsin-Madison

The past two decades have seen great strides in the mathematical training of chemical engineering graduate students, with impetus coming from the shift in the curriculum towards engineering science. Functional analysis, once completely absent from the chemical engineering literature, has become an important research tool for both analytical and numerical analyses. This new text provides an introduction to functional analysis with emphasis on analytical applications, at a level suitable for those students who have mastered the introductory engineering math course. This book was used as a required text in a one-semester course at the University of Wisconsin and this review is based on that experience.

Most math departments place a course in real analysis as a prerequisite for functional analysis. The first two chapters (labeled 0 and 1) of the book take on the difficult task of condensing the elements of real analysis into twenty-three pages, which is then followed by a chapter on linear spaces, Linear algebra as described in Chapter 2 is likely to be more abstract than that encountered in previous courses, but the approach sets the tone for later materials.

The following concepts are introduced in Chapter 3, "Metric Spaces": the metric, convergence of sequences, Cauchy sequences, continuity of functions, interior points, open and closed sets, limit points, closure of a set, compact sets, complete metric spaces, dense sets, etc. For the uninitiated, the information overload reaches a peak here, and since this portion of the course generally coincides with drop week, Chapter 3 represents the greatest challenge to both instructor and student.

Chapter 4 introduces Lebesgue integration and the elements of measure theory. This exposition achieves two results: it introduces an important example of completion of a metric space (space of Lebesgue integrable functions vs. Riemann integrable functions), and it prepares important examples of Banach and Hilbert spaces that appear in later chapters.

Chapters 5 and 6 cover normed linear spaces and inner product spaces respectively. This division facilitates the presentation of concepts that require only the norm instead of the full machinery of an inner product. The natural metric induced by the norm, the natural norm induced by the inner product, norm of an operator, compact operators, Banach spaces, Hilbert spaces, the Riesz Representation Theorem and adjoint of an operator are presented in these two chapters.

Although an exact demarcation is not possible, Chapters 0 through 6 may be viewed as preparation for applications found in later chapters. Chapter 7 features a rigorous derivation of the spectral theorem for both finite and infinite-dimensional Hilbert spaces. The derivation is essentially complete (back in Chapter 6 the authors chose to omit one step in the derivation of the projection theorem). Chapter 8 presents applications in finite-dimensional spaces such as multicomponent distillation problems.

In Chapters 9 and 10 (ODEs and PDEs respectively), the conversion of self-adjoint differential operators into integral equations via Green's functions is discussed in the context of the construction of compact inverses. Here we see the crucial distinction between a course based on *Linear Operator Methods in Chemical Engineering* and the standard graduate level engineering math course. In the latter, the instructor must resort to handwaving arguments when presenting the advantage of integral equations over differential equations. In the former, the advantage is rigorously self-evident. The reviewer was somewhat disappointed that a discussion on discrete spectra for infinite-interval problems was omitted in Chapter 9 (Titchmarsh is referenced). Perhaps this could be added in a later edition. The book ends with an introduction to non-self-adjoint operators and biorthogonal expansions (Chapter 11).

The authors have included enough material for a two-semester course, including many thought-provoking exercises. Our one-semester course covered all of Chapters 1, 2, 3, 5, 6, 7, 9, 10 and 11. This syllabus was intentionally weighted in favor of foundation over applications. A more leisurely pace can be set by reducing the emphasis on Chapters 1, 2, and 3. Other combinations are mentioned in the Preface.

In summary, this book is an excellent introduction to functional analysis and linear operator theory. The numerous "Chemical Engineering" applications make this book especially suitable for self-study. Those with "mathematical modeling" in their dissertation research should add this book to their personal library. □