A course in . . .

TRANSPORT PHENOMENA

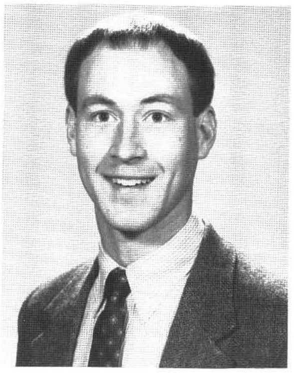
MARK J. McCREADY, DAVID T. LEIGHTON
University of Notre Dame
Notre Dame, IN 46556

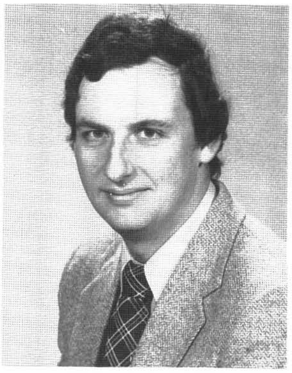
IF THERE IS ONE subject in which the philosophy of undergraduate instruction at various institutions could be best described as diverse, it is transport phenomena. Topics which fall under this heading may be found in courses titled as unit operations, fluid mechanics, heat and mass tansfer, or simply transport phenomena. The content of these courses is as varied as the titles are, with the resulting extremes being students who are either quite knowledgeable in the workings of various pieces of process equipment or who have a grasp of transport processes only on a microscopic level. Consequently, each student entering our graduate program has a different level of understanding of the basic principles governing the transport of heat, mass, and momentum as well as a diversity of the analytical skills which are necessary to solve these problems. The question becomes: How does one teach a single course sequence which all of these students will find interesting and challenging?

At Notre Dame this is done by following a philosophy for a two semester graduate transport phenomena course sequence which we suspect is similar to most other schools. The fundamental principles are explained and emphasized a number of times throughout the course. The skills necessary to solve the requisite differential equations are honed, and a significant amount of time is spent discussing example problems which display both important physical situations and interesting solution techniques.

The principal difference between our courses and those which we have encountered elsewhere is that we have designed the content and order of presentation so as to avoid placing undue hardships on students whose undergraduate education did not emphasize the formulation and solution of partial differential equations. This is done by saving most of the advanced mathematics for the second semester.

The first course strongly stresses the pertinent physics and the correct way to approach an arbitrary new problem, be it micro or macroscopic. When students learn some of the more powerful mathematical techniques for solving problems in heat and mass transfer in the second semester, they are able to explore problems involving greater mathematical complexity (such as Rayleigh-Benard convection and fluid flow past a heated sphere) without becoming overwhelmed.





Mark J. McCready joined the faculty at Notre Dame as an assistant professor after receiving a BChE degree from the University of Delaware and his MS and PhD degrees from the University of Illinois. His research interests lie in the area of fluid mechanics and transport properties of multiphase flows. Current topics include linear and nonlinear wave phenomena. **(L)**

David Leighton is an assistant professor in chemical engineering at the University of Notre Dame. After receiving his doctoral degree from Stanford University in 1985 he was a NATO Postdoctoral Fellow in the Department of Applied Mathematics of the University of Cambridge. His research interests at Notre Dame center on the study of the dynamic properties of sheared suspensions. **(R)**

FLUID MECHANICS

The subject of the "Transport Phenomena I" course, which is taught in the fall, is primarily fluid mechanics. In fact, given both of the instructors' research interests, the course could be better titled "Fluid Mechanics." A quick survey of simple macroscopic problems is done so that students who spent their summer in Europe or spinning discs at local dance establishments can reorient themselves to coursework. A homework problem set assigned the first class day includes both easy and difficult problems which are typically discussed in undergraduate courses. From the various complaints, it is possible to judge what topics must be reviewed. (It is interesting to note how many students have difficulty getting the correct *number* for pressure drop for turbulent flow in a smooth pipe.)

Lectures begin with a discussion of the kinds of forces which are found in fluid flows and how to describe them mathematically. The stress and strain tensors are introduced along with transformations and index notation. The primary references for this material are the texts by Whitaker [1] and Batchelor [2]. The boundary conditions which arise in various physical situations are then introduced. At this point it is possible to derive the mass and momentum conservation equations. The derivation is done both by shell balance and by using the substantial derivative to convert Newton's second law from a Lagrangian to an Eulerian framework. The conditions under which these equations reduce to the Navier-Stokes equations are examined.

The equations of motion are then used to solve problems in one or more dimensions, first for cases where exact solutions exist. Mathematical techniques such as separation of variables and special functions, which may be new to many students, are introduced in lectures and are used for homework problems.

A quick survey of the kind of fluid flow problems which engineers with advanced degrees may need to solve during their career indicates that not all of them should be approached from the microscopic view. Unfortunately, many students have gotten the idea that the macroscopic momentum equations are useful only to solve homework problems in undergraduate courses; they have the mistaken impression that all *real* problems will yield to a detailed analysis using the Navier-Stokes equations. In addition to not realizing whether differential or integral balances are appropriate, their ability to successfully apply integral balances to other than one dimensional problems is generally limited.

For this reason, lectures which deal with macroscopic problems are inserted at this point. Macroscopic balance equations are derived from the differential equations by the application of the divergence theorem and also by using integral averages of flows and forces on macroscopic control volumes. A typical homework problem might be the derivation of Dressler's equations for flow of a turbulent fluid in channel including the effects of air shear and surface tension. The mechanical "energy balance" is derived from the momentum balance for two reasons. The important concept of dissipation, which accounts for the missing energy, is introduced and the natural link between thermodynamics and fluid mechanics is developed.

This link is further explored when the next subject, compressible flow, is discussed. Compressible flows occur in numerous physical situations which chemical engineers may encounter, but they seldom receive much attention in courses. (Does the velocity of a gas really *increase* as it flows through a pipe? Why is the gas pump for my experiment not working at its rated flow rate?) When the macroscopic balance equation for total energy is derived and compared to the mechanical energy equation, the physical significance of dissipation in terms of entropy becomes clear. The relation between entropy production and velocity gradients is discussed. The concept of sonic velocity and choking are also introduced.

The focus of the course now shifts to follow what is more commonly taught in graduate transport courses—application of the Navier-Stokes equations to problems where exact solutions do not exist. Creeping flow is done first. The important idea here is that various nonzero terms are neglected not simply because they are small, but small in comparison to other terms. The physics of creeping flows is discussed in detail—what does it really mean to have no inertia? It is noted that velocity fields (solutions to Stokes' equations) are superimposable as a consequence of the linearity of the equations.

. . . we have designed the content and order of presentation so as to avoid placing undue hardships on students whose undergraduate education did not emphasize the formulation and solution of partial differential equations.

The solution to the zero Reynolds number equations is done for the sphere, and arguments leading to Stokes' paradox are investigated. The Oseen solution is done and Whitehead's paradox is discussed. At this point the general idea of perturbation solutions is introduced and used to improve the solutions for rotating flows. In addition, the matched asymptotic solution for flow around a sphere is briefly outlined.

The next topic, ideal fluid flow, commences with a description of the physical meaning of irrotationality and situations where it provides an accurate description. The primary source of information for lectures on ideal fluids is gotten from the texts by Streeter [3] and Lamb [4]. The velocity potential function is introduced and used to show that LaPlace's equation governs these flows. This leads to the amazing realization that velocity fields are superimposable for ideal flows as a consequence of the absence of shear forces even though the underlying Navier-Stokes equations are nonlinear. The solution of the problem for flow around a sphere leads to d'Alembert's paradox. The idea of circulation is introduced and the application of ideal fluid theory to the calculation of lift for flow over various bodies (including baseballs, golf balls, and sails) is discussed.

The next topic is boundary layer theory, for which Schlichting's text [5] serves as an invaluable reference. BLT is an especially rich subject to study in a graduate course because all of the many approximations which arise can be shown to follow as obvious consequences of Prandtl's observation that viscous and inertial forces balance near solid boundaries. To show the utility and validity of the various assumptions it is instructive to compare calculations of drag from solutions to the boundary layer equations with data to demonstrate that the approximations do in fact lead to good agreement.

Tests, of which there are usually two in addition to a final, are designed so that students may be creative as well as display a basic level of understanding of the course material. Questions from the most recent semester included the locomotion of cephalopods, flow over porous airplane wings, and wave propagation described by an Orr-Sommerfeld equation. On each test, problems which require either integral or differential balances are interspersed. This requires that students think about which approach is appropriate. After the first course, students are expected to ask the correct questions when confronted with a new problem. They should know how to examine the essential physics on an appropriate scale.

HEAT AND MASS TRANSFER

In the spring, the topics switch to heat and mass transfer. Up to this point the time spent examining macroscopic problems and emphasizing the physics for each situation has limited the number of important analytical methods the students have been exposed to and which may be necessary for solving difficult detailed problems that arise in their research. In the second semester the emphasis on physical principles is retained, but the problems discussed also serve to introduce the students to advanced mathematical techniques.

As in the first semester, the second semester begins with the derivation of the transport equations—this time energy transport—only now the equations are derived using vector notation. A detailed understanding of how the equation of energy works in vector form is built by asigning problems such as the derivation of the rate of entropy production. The text for this material (in addition to the texts used in the first semester) is Bird, Stewart and Lightfoot [6]. Einstein notation is also re-introduced at this point in the course (as it was not used extensively in the first semester), leading to a great simplification in the form of the transport equations.

Following a conventional sequence, steady conduction in solids is reviewed, first assuming constant properties and then relaxing this restriction to include non-constant properties, introducing the student to regular perturbation methods. A supplemental text for perturbation methods is Van Dyke [7], which is further utilized when matched asymptotic expansions are discussed later in the course. The course now turns to the effects of convective energy transport, examining problems such as transpirational cooling and forced convection through a heated pipe. Rather than using a cookbook approach to the Graetz problem, the students are introduced to the formal theory of a Sturm-Liouville eigenvalue problem. Particular emphasis is placed on when to expect this type of solution and how to cast the problem into the Sturm-Liouville form.

Dimensional analysis is the next topic of discussion. However, here we differ from the usual transport class in that dimensional analysis is introduced in the context of the large field of similitude. The references for this material are the notes from a course on similitude taught by Van Dyke [8] which are distributed to the class. Over one week is spent introducing the students to techniques for finding hidden symmetry in physical problems, first through the use of dimensional and inspectional analysis for the reduction of the number of independent parameters involved in a problem and then via more advanced techniques, such as coordinate stretching to achieve reductions in the number of independent variables upon which a problem depends. These techniques are illustrated by examples from both momentum and energy transport, such as the determination of the radius of a shock wave produced by an intense point explosion solved by G. I. Taylor [9], the velocity field of a submerged laminar jet, and such whimsical examples as the spread of a viscous thread of liquid flowing down an inclined plane.

The concept of self-similarity is put to immediate use in the next topic—that of unsteady conduction in solids. In addition to the standard semi-infinite and finite slab problems, a semi-infinite slab with a melting boundary is also discussed. Students are asked to explain why such a problem with a step change in temperature at the edge of the slab admits a similarity solution, but such a solution for a constant heat flux does not exist.

The course next turns to boundary layer theory for forced convection past a heated, horizontal flat plate. This problem is solved in the limiting cases of large and small Prandtl numbers, and then the plate is turned to the vertical for a discussion of free convection. For homework, students use the concept of self-similarity to solve the analogous problem of a free-convection laminar jet arising from a point source of energy. A general dimensional analysis of the free and forced convection transport equations is inserted at this point so that the students can develop an intuitive feel for the relative magnitude of the two transport mechanisms.

The study of free convection is continued by examination of the instability of a fluid heated from below. The Rayleigh-Benard stability problem for free-free boundaries is discussed in detail, the reference for this discussion being the text on hydrodynamic stability by Drazin and Reid [10]. The students' understanding of the principles of this mathematically complex phenomenon is reinforced by homework in which the stability conditions for problems analogous to the Rayleigh-Benard problem are worked out and also by assignments on a more cosmic scale in which the students solve the Jeans problem for the gravitational collapse of a galactic sized gas cloud.

At this point in the course we begin our discussion of singular perturbation theory, drawing heavily on the text by Van Dyke. First, we examine the classic problem of creeping flow past a heated sphere at small Peclet number solved by Taylor and Acrivos [11]. This problem serves to introduce the concept of a non-uniformly valid first approximation, and why a regular perturbation approach to such problems is doomed to failure. The students are shown how to overcome these difficulties via a matched asymptotic expansion approach which, in this problem, also introduces the student to special mathematical functions such as spherical harmonics and Legendre polynomials. Flow past a sphere is followed up by such problems as flow through a tube with an axial wire and unsteady conduction from an infinite cylinder. The method of reflections comes next, in which we emphasize the similarity of this technique to the singular perturbation methods just discussed and which is used to determine the energy loss from a heated sphere in the vicinity of a plane. The analogous problem of a heated cylinder near a plane, which cannot be solved using perturbation techniques, is also examined and solved using conformal mapping, adding yet another technique for obtaining solutions of the transport equations to the students' arsenal.

Brief discussions of turbulent and radiative transport mechanisms complete the portion of the course dealing with energy transport. Topics discussed here include Prandtl mixing length theory and transport correlations in turbulent systems, together with the concepts of isotropy, black and gray bodies, view factors, an introduction to configurational algebra and spectral effects in radiative energy transport.

With three weeks remaining, the course turns towards mass transport. The first two lectures are devoted to definitions, the description of mass transport in terms of Fick's Law, and derivation of the transport equations. Simple problems come next, such as the Stefan tube and diffusion with homogeneous or heterogeneous chemical reaction (the Thiele problem). Combined mass, momentum and energy transport in boundary layers is discussed in which the effect of mass transport on the evolution of the thermal and momentum boundary layers is examined. Students are also exposed to mass transport mechanisms not usually encountered in undergraduate courses, such as pressure diffusion, forced diffusion (electrophoresis), and the Soret effect. In a typical problem at this point, students are asked to analyze a Clusius-Dickel column (a separations device which relies on the Soret effect), where they are required to determine what assumptions are necessary to obtain a solution.

The last formal topic discussed in the course involves the unsteady one- and two-dimensional diffusion of a trace pollutant, focusing on problems such as the steady or unsteady discharge from a waste pipe into a stream. The similarity between pollutant concentration distributions resulting from the unsteady convective diffusion equation and a probability distribution arising from stochastic differential equations is emphasized. The final two lectures are devoted to research interests: one lecture by a professor whose research is in the area of energy or mass transport, and one lecture by a student in the class working in the same area who by this time is getting ready for the first year comprehensive oral examination. These last lectures give the students some feel for the utility of the topics and techniques discussed during the semester in the solution of current graduate research problems.

In conclusion, this course sequence is designed to meet the needs of students from diverse backgrounds who enter our graduate program. Students are first introduced to the governing physics without undue emphasis on mathematical techniques. As their level of understanding increases and their problem solving approach becomes better refined, more sophisticated techniques are introduced. When students have com-

Continued on page 218.

REVIEW: Injection Molding

Continued from page 173.

plains the methodology of process control. This chapter bears some similarity to Chapter Three, but is much more thorough and useful.

The last section (Part III) is concerned with data bases and contains Chapter Twelve. It is one of the more useful chapters in the book as it describes the importance to the designer of having data banks available containing the physical properties in both the solid and molten phases of each thermoplastic. This data should be readily available in both the part design and process simulation phases and must be stored in the computer system. The chapter contains an overview of the development of the present data bases, including the types of data available in present systems and future trends.

In summary, there are a number of useful chapters in the book, but unfortunately the connection between chapters is not readily apparent. For the inexperienced engineer, it would be difficult to assemble the appropriate knowledge from this book and then apply it to process control or mold design. The book would be more useful if a section on principles of injection molding, including the fluid mechanics of mold filling and its connection to the properties of a part, were included at the beginning of the book. □

TRANSPORT PHENOMENA

Continued from page 177.

pleted these courses they will know what to look for when they encounter new problems, and they will have acquired the tools necessary to solve a great many of them.

REFERENCES

1. Stephan Whitaker, *Introduction to Fluid Mechanics*, Krieger 1981.
2. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge, 1967.
3. V. L. Streeter, *Fluid Dynamics*, McGraw Hill, 1948.
4. Horace Lamb, *Hydrodynamics*, Cambridge, 1932.
5. Hermann Schlichting, *Boundary-Layer Theory*, McGraw-Hill, 1968.
6. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley, 1960.
7. Milton Van Dyke, *Perturbation Methods in Fluid Mechanics*, Parabolic, 1975.
8. Milton Van Dyke, Course Notes for ME 206, Similitude in Engineering Mechanics, January 1978.
9. G. I. Taylor, "The Formation of a Blast Wave by a Very Intense Explosion," *Proc. Roy. Soc. A.*, **201**, pp. 159-186.
10. P. G. Drazin and W. H. Reid, *Hydrodynamic Stability*, Cambridge, 1981.
11. Andreas Acrivos and G. I. Taylor, *Phys. Fluids*, **5**, p. 387 (1962). □

MICROGRAVITY

Continued from page 193.

gineers to develop entirely new processes, to understand current unit operations more thoroughly, or to adapt earth-based unit operations for the demanding environment of space.

REFERENCES

1. Allen, D. T., and D. Pettit, Symposium on Zero Gravity Processing, AIChE Spring National Meeting, Houston, 1985.
2. Subramanian, R. S., and R. Cole, Symposium on Transport Phenomena in Space Processing, AIChE Annual Meeting, New York, 1987.
3. NASA Technical Memorandum 89607, "Microgravity Science and Applications Program Tasks," NASA Office of Space Science and Applications, Washington D.C., February 1987.
4. NASA Technical Memorandum 89608, "Microgravity Science and Applications Bibliography," NASA Office of Space Science and Applications, Washington, D.C. January 1987.
5. Naumann, R. J., and H. W. Herring, "Materials Processing in Space: Early Experiments," NASA SP-443, 1980.
6. NASA Marshall Space Flight Center Publication, "Microgravity Science and Applications: Experimental Apparatus and Facilities," Washington, D.C.
7. Potard, C., and P. Dusserre, "Contactless Positioning, Manipulation and Shaping of Liquids by Gas Bearing for Microgravity Applications," *Adv. Space Res.*, **4**(5), 105-108 (1984).
8. Ray, C. S., and D. E. Day, "Description of the Containerless Melting of Glass in Low Gravity," *SAMPE Tech. Conf. Ser.*, **15**, 135 (1983).
9. Doremus, R. H., "Glass in Space," in *Materials Science in Space* (B. Feuerbacher *et al.*, Eds.), Springer-Verlag, 1986, p. 447.
10. Doremus, R. H., "Glass Shell Fabrication Possibilities as Viewed by a Glass Scientist," *J. Vac. Sci. Tech.*, **A3**, 1279 (1985).
11. Swanson, L. W., "Optimization of Low Gravity Float Zone Crystal Growth," M.S. Thesis, University of California, Los Angeles, 1983.
12. Naumann, R. J., Marshall Flight Center Space Science Laboratory Preprint Series No. 86-137, June 1986.
13. Shankar, N., and R. S. Subramanian, "The Slow Axisymmetric Thermocapillary Migration of an Eccentrically Placed Bubble inside a Drop in Zero Gravity," *J. Colloid Interface Science*, **94**, 258-275 (1983).
14. Snyder, R. S., P. H. Rhodes, T. Y. Miller, F. J. Micale, R. V. Mann, and G. V. F. Seaman, "Polystyrene Latex Separations by Continuous Flow Electrophoresis on the Space Shuttle," *Sep. Sci. Tech.*, **22**, 157-185 (1986).
15. Saville, D. A., and O. A. Palusinski, "The Theory of Electrophoretic Separations. I: Formulation of a Mathematical Model," *AIChE J.*, **32**, 207-214 (1986).
16. Saville, D. A., O. A. Palusinski, R. A. Graham, R. A. Mosher, and M. Bier, "The Theory of Electrophoretic Separations. II: Construction of a Numerical Simulation Scheme," *AIChE J.*, **32**, 215-223 (1986).
17. Sacco, A., L. S. Sand, D. Collette, K. Dieselman, J. Crowley, and A. Feitelberg, "Zeolite Crystal Growth in Space," AIChE Spring National Meeting, Houston, 1985.
18. Cherry, R. S., and E. T. Papoutsakis, "Hydrodynamic Effects on Cells in Agitated Tissue Culture Reactors," *Bioprocess Engr.*, **1**, 29-41 (1986). □