

A SIMPLER WAY TO TAME MULTIPLE-EFFECT EVAPORATORS

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METHODS FOR SOLVING multiple-effect evaporators fall into three broad classes: a) traditional trial-and-error taught in most textbooks [1, 2, 3], b) simplified methods used as first-order design approximations [4], and c) complex methods such as dynamic programming and evolutionary operations [5, 6, 7]. In the past, the only one of the above ever taught in the classroom (if evaporators were taught at all) was the traditional trial-and-error method. This method is extremely limited and almost useless for evaporator series

larger than three, yet virtually all unit operations textbooks discuss it in some detail. The simplified design methods are easy to use (assuming no boiling point rise), but are not flexible or particularly accurate; and the computer methods, although accurate and flexible, are generally much too sophisticated for effective teaching, especially if the engineering aspects of an evaporator train are to be taught as well.

In the classroom, the subject of evaporators is getting squeezed out of the undergraduate curriculum. This is partly the result of the difficulty in teaching it without getting hopelessly bogged down in the analysis. A new method is presented here, the details of which have been recently published elsewhere [8, 9], which is both simple enough to use in the classroom and accurate and flexible enough to be used as a design tool in practice.

HOW SHOULD EVAPORATOR SYSTEMS BE APPROACHED?

In searching for more efficient ways to solve the evaporator system, it becomes clear that a different approach than the three previously mentioned is required. The evaporator series can be viewed as another stagewise unit operation, where fundamental equations for heat and mass balances can be written for each stage, with the heat transfer rate equation substituting for an equilibrium relationship. These equations, when written in such a manner, are not easy to solve. The evaporator train equations, shown below for the backward-feed system illustrated in Figure 1, do not lend themselves to solving directly as a set of nonlinear algebraic equations. In fact, almost none of these kinds of nonlinear equations are easy to solve.

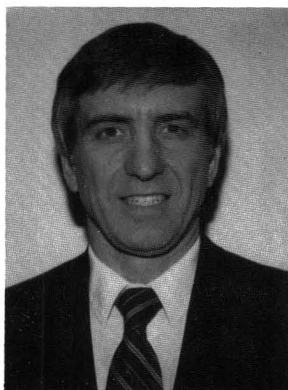
The equations for the i^{th} stage in backward feed are:

$$\text{mass balance} \quad L_{i+1} - L_i - V_i = 0 \quad (1)$$

$$\text{heat balance} \quad \Delta H(x, T)_{i-1} + h(x, T)L_{i+1} - H(x, T)V_i - h(x, T)L_i = 0 \quad (2)$$

$$\text{heat transfer rate} \quad [\lambda(T_s) + SH(x, T, T_s)]V_{i-1} = U_i(x, T, \dots)A_i(T_{si-1} - T_i) \quad (3)$$

The variable A_i in Eq. (3) can also be written as AR_i to allow for variable area situations, where R_i is the ratio of area of the i^{th} stage to A . Setting $R_i = 1.0$ gives the most common solution corresponding to equal-area evaporators, which simplifies to $A_i = A$.



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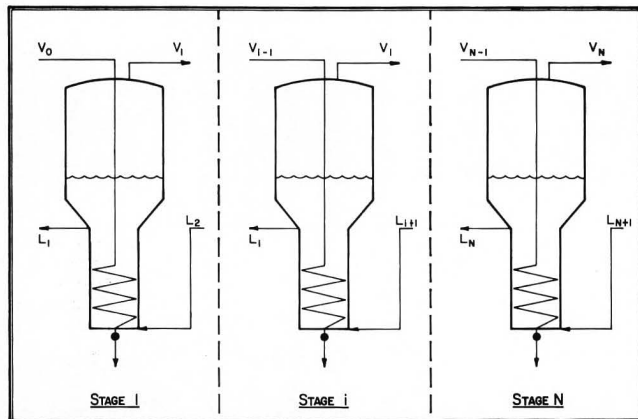


FIGURE 1. Backward feed, N-effect evaporator series

The variable T_{si-1} is the saturation temperature of the previous stage and can be computed as follows:

$$T_{si-1} = T_{i-1} - BPR_{i-1} \quad (4)$$

where BPR is the boiling point rise. The boiling point rise can be calculated from a knowledge of the solids concentration and a Dühring chart. The solids concentration can be computed from the solids balance

$$x_i L_i = \text{solids} = \text{constant} \quad (5)$$

where x is the solids concentration.

In Eqs. (1) through (5) standard symbols have been used for liquid (L) and vapor (V) flow rates, overall heat transfer coefficient (U), temperature (T), heat transfer area (A), liquid and vapor enthalpies (h and H , respectively) and heat of vaporization at the saturation temperature (λ). Subscript "s" is used to indicate saturated conditions, and the variable SH is the superheat in the vapor occurring as a result of a boiling point rise. The superheat is the enthalpy rise above saturated vapor enthalpy at constant pressure. Subscripting is arranged so that externally supplied steam always enters the first stage. At the ends of the series some of the variables are known. Corresponding equations for forward feed can be generated by simply changing the $i+1$ subscript to $i-1$ on all liquid flows (only). Mixed feed situations may be handled by writing all the equations explicitly to show the connections between stages.

The non-linearities in Eqs. (2) and (3) arise from two sources; the coefficients are non-linear functions of x and T , and in some cases other variables as indicated, and a cross-product term, AT , exists. The hurdle of non-linearity can be eliminated by redefining the cross-product term as a new

variable, AT . Then the set of equations is sequentially linear, *i.e.*, at any T_i the coefficients are defined. Thus the resulting equations can be put into matrix form and solved by any linear technique, such as Gaussian elimination. Of course, iterations will be required if the new values of T_i are significantly different from the old. Because the coefficients are not strong functions of T , the equations don't change very much from iteration to iteration, and convergence is very rapid.

For example, Eqs. (1), (2), and (3) are rewritten below, incorporating the changes discussed above, for a constant-area solution.

$$(-1)L_i + (-1)V_i + (1)L_{i+1} = 0 \quad (6)$$

$$aV_{i-1} + bL_i + cV_i + dL_{i+1} = 0 \quad (7)$$

$$eV_{i-1} + fAT_{i-1} + gAT_i + kA = 0 \quad (8)$$

where the coefficients a-k are

$$a = \Delta H(x, T)_{i-1} = [\lambda(T_s) + SH(x, T, T_s)]_{i-1}$$

$$b = -h(x, T)_i$$

$$c = -H(x, T)_i$$

$$d = h(x, T)_{i+1}$$

$$e = [\lambda(T_s) + SH(x, T, T_s)]_{i-1}$$

$$f = -U_i$$

$$g = -f = U_i$$

$$k = -U_i BPR_{i-1}$$

In general there are $3N + 4$ unknowns and $3N$ equations. Four boundary conditions can be specified, for example:

$$L_{N+1} = F = \text{feed rate}$$

$$L_1 = P = \text{product rate}$$

$$AT_0 = G_1 A, \quad \text{thus} \quad G_1 = T_0$$

$$AT_N = G_2 A, \quad \text{and} \quad G_2 = T_N$$

This results in a set of $3N + 4$ linear, algebraic equations with constant coefficients, which can be solved easily once the values for the coefficients are known.

The method of solution, described in detail elsewhere [8, 9], can be summarized by the following:

- Write the governing equations in linear format
- Initialize temperature and composition in each effect by apportioning the vapor flows and temperature drops

- Find the coefficients of the variables. These coefficients are the enthalpy values, boiling point rises, and overall heat transfer coefficients, all of which are functions of temperature and composition, or are specified as design (input) parameters
- Solve the matrix of coefficients by any linear technique to obtain values for the variables (L , V , AT and A) for each stage
- Compute $T_i (= AT_i/A)$ and $x_i (= x_N L_N/L_i)$
- When new values of T and x lead to significant changes in the coefficients, do another iteration

Initialization to get first values for the coefficients may be done by any method, including guessing, but the traditional method of apportioning vapor flows and T 's leads to more rapid convergence. This is especially important when doing this by hand calculation but less important when a fully computerized method is used.

EXAMPLE

The following example shows how students can solve a triple-effect evaporator series easily, if they have enthalpy tables, a Dühring chart, and a straightforward linear equation solver. Students were asked to find the steam rate and the area for one, two, and three-stage systems given the following information. The feed was aqueous sodium hydroxide.

$$\begin{aligned} \text{Feed} &= 20,000 \text{ kg/hr} \\ x_F &= 0.05 \\ x_P &= 0.60 \\ T_F &= 3^\circ\text{C} \\ T_0 &= 163^\circ\text{C, saturated steam} \\ p_N &= 17270 \text{ Pa} \\ U_N &= 787.8 \text{ W/m}^2\cdot\text{K} = 2836 \text{ kJ/hr}\cdot\text{m}^2\cdot\text{K} \quad (\text{when } N=2,3) \\ U_1 &= 157.8 \text{ W/m}^2\cdot\text{K} = 567 \text{ kJ/hr}\cdot\text{m}^2\cdot\text{K} \\ U_{\text{mid}} &= 1701 \text{ kJ/hr}\cdot\text{m}^2\cdot\text{K} \quad \text{for middle stage when } N=3 \end{aligned}$$

The units of U are changed to be consistent with enthalpy units. Equations 2 and 3 can then be rewritten in a form to make the coefficients and variables more obvious.

$$hL_i + (H_s + SH)V_i - hL_{i+1} - (\lambda + SH)V_{i-1} = 0 \quad (6)$$

$$U_i(AT_{i-1}) - U_i BPR_{i-1}(A) - U_i(AT_i) - (\lambda + SH)V_{i-1} = 0 \quad (7)$$

Here H_s is the vapor enthalpy at saturated conditions.

Preliminary calculations for the boundary conditions and initial estimates of internal temperatures and flows are shown below.

$$P = x_F F / x_P = 0.05(20000)/0.60 = 1667$$

$$\begin{aligned} V_{\text{tot}} &= 20000 - 1667 = 18333 \\ V_1 &= V_2 = 9166.7 \\ x_2 &= x_F F / (F - V_2) = 0.09 \\ BPR_2 &= 2^\circ\text{C} \quad (\text{from Dühring chart}) \\ BPR_1 &= 60^\circ\text{C} \quad (\text{from Dühring chart}) \\ T_F &= 3^\circ\text{C} \\ T_0 &= 163^\circ\text{C} \\ T_{s2} &= 57^\circ\text{C} \quad (\text{from } p_2 = 17.27 \text{ kPa}) \\ \Delta T_{\text{tot}} &= T_0 - T_{s2} - BPR_1 - BPR_2 = 44^\circ\text{C} \\ U_{\text{tot}} &= 1/(1/U_1 + 1/U_2) = 1/(1/567 + 1/2836) \\ &= 472.5 \text{ kJ/hr}\cdot\text{m}^2\cdot\text{K} \\ \Delta T_1 &= \Delta T_{\text{tot}}(U_{\text{tot}}/U_1) = 36.7^\circ\text{C} \\ &\quad \text{thus } T_1 = 126.3, \quad T_{s1} = 66.3 \\ \Delta T_2 &= \Delta T_{\text{tot}}(U_{\text{tot}}/U_2) = 7.3^\circ\text{C} \\ &\quad \text{thus } T_2 = 59, \quad T_{s2} = 57 \end{aligned}$$

One can see that the equal boilup assumption was used for the calculation of the boiling point rise, and the driving force temperature differences were apportioned by inverse U relationship. These are the elements of the traditional startup. The coefficients for the stage equations can now be calculated and entered into matrix form according to Table 1. The order of the columns should always be consistent to minimize confusion in a hand calculation. The last four rows need not be solved as part of the matrix, but the calculations are more convenient to do in the matrix than outside it. When additional stages are to be investigated, the new rows and columns can be entered as indicated in Table 1.

The above procedure is done to obtain a square matrix of coefficients with dimension $3N + 4$. Any linear technique can be used to solve this easily with a given right-hand-side vector. For example, an in-house package developed by Koko was used at Bucknell, and PC-MATLAB was used at Villanova. The IMSL package has been used successfully at both places, and student-programmed Gaussian elimination can also be used, but the packages are very convenient. These methods have worked in systems with up to ten stages with no major difficulty.

The matrix of coefficients for the two-stage, backward feed problem is shown below. The coefficients of L and V in Eq. (1) are always 1 or -1 , thus only the coefficients of Eq. (2) and (3) need be calculated.

$$\begin{aligned} h_2 &= 221 \text{ kJ/kg} \quad (\text{at } 59^\circ\text{C} \text{ and } 0.09) \\ h_3 &= 12 \text{ kJ/kg} \quad (\text{at } 3^\circ\text{C} \text{ and } 0.05) \\ H_1 &= H_{s1} + SH_1 = 2619 + 1.88(60) = 2732 \text{ kJ/kg} \\ H_2 &= H_{s2} + SH_2 = 2604 + 1.88(2) = 2608 \text{ kJ/kg} \\ \lambda_0 &= 2073 \text{ kJ/kg} \quad (\text{at } 163^\circ\text{C}) \end{aligned}$$

$$h_1 = 819 \text{ kJ/kg (at } 126.3^\circ\text{C and } 0.60)$$

$$\lambda_1 = 2345 \text{ kJ/kg (at } 66.3^\circ\text{C)}$$

$$(\lambda + SH)_1 = 2345 + 1.88(60) = 2458 \text{ kJ/kg}$$

$$(U_2 BPR_1) = (2836)(60) = 170160 \text{ kJ/hr}\cdot\text{m}^2$$

The U 's and BPR 's are known from the preliminary calculations. For calculation of superheat values, heat capacity of water vapor was taken as $1.88 \text{ kJ/kg}\cdot\text{K}$. Liquid enthalpies were obtained from sodium hydroxide/water data [1, 2, 3], and vapor enthalpies and heats of vaporization were obtained from saturated steam tables [1, 3, 4].

The matrix is then solved by a linear equation solver. The answers for the first iteration are shown in Table 1, below the matrix. One can see that these values are very close to the guesses, *i.e.* $T_1 = 125.9$ compared to the guessed value of 126.3 . Thus, for a hand calculation, one iteration has been sufficient.

HOW GOOD IS THIS APPROACH?

This approach yields an algorithm that can be made extremely flexible and accurate. By far the most attractive feature, besides its simplicity, is its inherent stability when fully computerized.

Students have used the hand calculation method successfully, and they agreed that this method was much easier to use and to understand than the traditional trial-and-error method. We provided students with a simple computerized linear algebraic equation solver that allowed changing indi-

vidual coefficients and re-solving the matrix. We found that at least two hours of lecture were required to present the concepts of an evaporator series, the balance equations, and the tableau into which they had to place the coefficients. The hardest concepts for junior chemical engineers to understand were boiling point rise and the effect of superheat, apportioning the available ΔT for the initial guess, and getting the correct enthalpies when significant heat of solution effects were present.

In the fully computerized method reported in detail elsewhere [8, 9], we found that convergence could be monitored by any single-valued function, such as the steam rate. The fully computerized method converged from any starting point and was at least an order of magnitude faster than nonlinear equation solving techniques, if the latter converged at all. The algorithm did not need a traditional equal boilup start and easily handled 30-stage forward and backward-feed systems with complex liquid behavior like that exhibited by sodium hydroxide and water mixtures.

In addition, the computer routine can be incorporated into a search program to find an economic optimum design. The ratio of areas, R_i , may come into play here, if area were to be used to compensate for varying U , for example, or BPR changes.

WHAT PROCESS INFORMATION CAN BE EASILY SEEN?

The algorithm solves for the classical variables of interest, *viz.*, liquid and vapor flow rates, the temperature and composition in each stage, the size of an individual

TABLE 1
Matrix of Coefficients for the 2-Stage,
Backward Feed, Sodium Hydroxide Problem

		order of variables					(higher stage variables inserted here)					
		\underline{AT}_0	\underline{V}_0	\underline{L}_1	\underline{AT}_1	\underline{V}_1	\underline{L}_2	\underline{AT}_2	\underline{V}_2	\underline{L}_3	\underline{A}	\underline{RHS}
$i = 1$	}	0	0	-1	0	-1	1	0	0	0	0	0
		0	-2073	819	0	2732	-221	0	0	0	0	0
		567	-2073	0	-567	0	0	0	0	0	0	0
$i = 2$	}	0	0	0	0	0	-1	0	-1	1	0	0
		0	0	0	0	-2458	221	0	2608	-12	0	0
		0	0	0	2836	-2458	0	-2836	0	0	-170160	0
		(higher stage equations inserted here)										
boundary conditions	}	0	0	1	0	0	0	0	0	0	0	1667
		0	0	0	0	0	0	0	0	1	0	20000
		1	0	0	0	0	0	0	0	0	-163	0
		0	0	0	0	0	0	1	0	0	-59	0
Solution		211800	13360	1670	162920	10240	11910	76660	8090	20000	1300	—
		$\underline{T}_0 = 163$			$\underline{T}_1 = 125.9$		$\underline{X}_2 = .086$	$\underline{T}_2 = 58.9$				

evaporator (heat transfer area) and the required flow rate of externally supplied steam. Practical consequences of the design can also be investigated by looking at other variables not normally of interest, such as heat transferred in each effect, pressure in each effect, etc.

Using the computer program, two design problems were made very apparent. One is what we term "boiling point rise failure," and the other is what we term "sensible heat demand failure." In systems with substantial boiling point rise, as the number of stages increases the sum of the boiling point rises also increases. It can increase to the point of exceeding the overall temperature driving force (the temperature difference between externally supplied steam and final effect saturation temperature), in which case the system breaks down [1]. The output of our algorithm showed a negative area appearing when this boiling point rise failure occurred. This is likely to happen in systems having a significant boiling point rise, such as sodium hydroxide/water, when the number of stages is large. The maximum number of stages where this occurs cannot be computed easily by hand, but the computer method can do evaluations of all cases from 1 to N stages, from which such information can be readily seen.

A sensible heat demand failure occurs in a stage when the incoming steam gives up all its heat to raise the temperature of the incoming liquid to its boiling point, and none is left over to evaporate the liquid. This can happen in the last stage (feed stage) of a backward-feed system, but it cannot happen in a forward-feed system [9], because temperature is highest in the first (feed) stage and drops in each successive stage. This condition has not been generally recognized as a cause for failure in evaporator systems, but it can occur with fewer stages than would cause a boiling point rise failure in backward-feed systems with a large boiling point rise. As the cost of energy increases, evaporator trains with more stages will be economical, and both failure modes could become even more important. One cannot have an arbitrarily large number of stages in backward feed situations, or in systems with a large boiling point rise. Some evaporator systems, therefore, cannot have unlimited economy, the ratio of total vapor evaporated to externally supplied steam.

WHAT OTHER KINDS OF PROBLEMS CAN IT SOLVE?

The flexibility of the computer method is such that different combinations of independent and dependent variables can be used. For example, one could have A as an independent variable and solve for p_N (the pressure in the last stage).

Also, functional dependencies of some of these variables relative to A may be of some interest. One could use the fully computerized model to find p_N as a function of A by selecting several values of p_N and solving for corresponding values of A, plotting these and backing out p_N as a function of A. In a similar manner one could back out N, the number of effects, given A, and so on. Thus, this algorithm can be used to calculate how many effects of a given size would be required to do a given evaporation problem.

The model could also be used as the performance function of a non-linear optimization program in which the independent variables, including R_i 's, are treated as the design or search variables and some estimate of overall cost (capital plus operating costs) is used as the performance function. If pricing functions for T_0 and p_N are available, they could be included as search variables for the economic design.

WHY SHOULD I USE THIS METHOD?

In conclusion, the reasons for using this approach are

- *The functional form of the equations is readily apparent. The method can be taught easily and does not get much more complicated as the number of stages increases beyond two.*
- *Non-linearities in enthalpy or boiling point rise are inconsequential to the algorithm and easily incorporated.*
- *Physical limitations on the system can be readily identified.*
- *The solution scheme involves simple numerical methods, such as successive substitution with Gaussian elimination for the solution of the simultaneous linear algebraic equations at each iteration. Small-N problems done by hand rarely need a second iteration.*
- *The algorithm may be used in complex economic optimization routines. The method encourages investigating other relationships in the evaporator system.*
- *The fully computerized algorithm is unusually efficient and stable, with very favorable convergence characteristics shown even where non-linear methods fail to converge.*
- *This approach fits in well with computer methods for staged operations and may be very useful in that area, because it shows how some non-linearities can be handled simply and efficiently.*

The potential usefulness of this method is exciting because, (a) it is easy to teach as our experience has shown, (b) it inspires investigation of other evaporator problems previously ignored as too difficult or not worth the trouble, and (c) it might have a general application in other unit operations problems, such as absorption and non-ideal distillation analysis.

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