This one-page column will present practical teaching tips in sufficient detail that ChE educators can adopt the tip. The focus should be on the teaching method, not content. With no tables or figures the column should be approximately 450 words. If graphics are included, the length needs to be reduced. Tips that are too long will be edited to fit on one page. Please submit a Word file to Phil Wankat <wankat@ecn.purdue.edu>, subject: CEE Teaching Tip.

EXPLAINING THE CONVECTIVE TERM IN THE NAVIER-STOKES EQUATIONS

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Many undergraduates struggle with the shell balance approach to deriving conservation equations in transport phenomena, in particular the convective term ($\rho v_x v_x$) of the Navier-Stokes momentum equation. For simplicity and brevity we will focus on the x-component in a unidirectional flow system in rectangular coordinates given by:

$$\frac{\partial(\rho v_x)}{\partial t}+\frac{\partial(\rho v_x v_x)}{\partial x}=-\frac{\partial \tau_{xx}}{\partial x}-\frac{\partial P}{\partial x}+\rho g_x \qquad (1)$$

Students have difficulty comprehending what is done in the leading textbooks,[1-4] where it is stated that the rate of momentum entering the element is $\rho v_x v_x$. The shell balance approach is typically followed with the student question "Why is that term rho-v-v?" Providing the typical answer that "the units work out" is not an explanation.

DESCRIPTION OF THE METHOD

The following physical derivation for the bulk flow or convective term is easier for students to understand. We begin our analysis by considering a stream of elements, of width w, flowing into a control volume $x\ y\ z$, as illustrated in Figure 1. These elements bring with them momentum due to their mass and velocity.

The mass of each element is $\rho V = \rho w\ y\ z$. The momentum is given by mv_x, which is $\rho v_x w\ y\ z$. The rate at which n elements (they are in a continuously moving stream) come into the control volume is $n/\ t = v_x\ /w$. Thus, the rate of momentum brought into the control volume is equal to the product of the momentum of each element multiplied by their rate: $\rho v_x w\ y\ z v_x\ /w = \rho v_x v_x\ y\ z$. Subtracting this value from the momentum going out of the control volume, dividing by the control volume, and taking the limit as $x \rightarrow 0$, gives $\frac{\partial(\rho v_x v_x)}{\partial x}$.

GENERALIZATION OF THE METHOD

This method can be generalized. For instance, the energy entering the volume element of width w is $\rho C_p T w\ y\ z$. Following the above procedure gives rise to the thermal energy convection term, $\frac{\partial(\rho C_p v_x T)}{\partial x}$.

Also, using the species mass $C_A w\ y\ z$ entering the control volume gives rise to the mass convection term, $\frac{\partial(v_x C_A)}{\partial x}$.

CONCLUSIONS

Physical intuition is used to derive the convective terms in chemical engineering transport equations. Using this method will allow students to better understand the nature of convective transport governing many applications.

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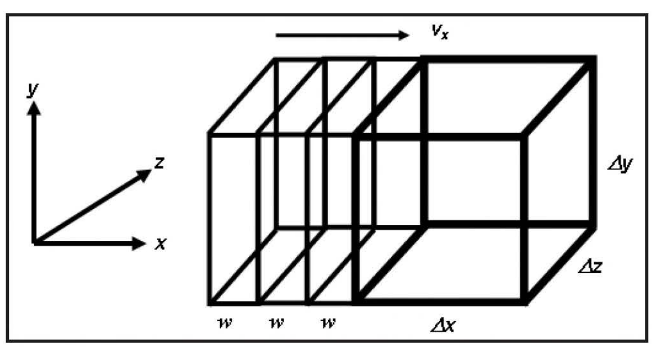

Figure 1. Conceptual cartoon of bulk flow into a control volume (thicker lines).